How important is Belief Heterogeneity of Households?

Kwok Yan Chiu*

August 2023

Abstract

Macroeconomic expectations are known to correlate with socioeconomic status, but this relationship is absent in most heterogeneous-agent models. I find that, specifically, households with low marginal propensities to consume (MPC) or high elasticity of intertemporal substitution (EIS) update their forecasts faster than others in response to the business cycle. I develop and estimate a heterogeneous-agent model with rational expectations that captures the empirical correlation between beliefs and household characteristics. Compared to a typical calibration that assumes no such correlation, I find that this model implies more amplification and consumption heterogeneity in response to shocks.

[Very Preliminary. Do Not Cite or Distribute.]

*Northwestern University; KwokYan.Chiu@u.northwestern.edu.
I thank Alireza Tahbaz-Salehi, Matthias Doepke, Marios Angeletos, and Matt Rognile for their guidance and support. I also thank Matias Bayas-Erazo and Fergal Hanks, Michael Cai, Joao Guerreiro, and participants in Macroeconomic Lunch Seminars for helpful comments and suggestions.

1 Introduction

How important is belief heterogeneity of households? The heterogeneity in macroeconomic expectation suggests that households may have different abilities to form expectations about the future. This can affect transmission of demand shock to individuals consumption. Literatures following Friedman (1957) and Modigliani (2005) show that individual consumption depends on their current and future expected discount rate shock, as well as their current and future expected personal income. Information affects households expectation about future shocks and income, and thus consumption decision.

The effect of information to consumption depends on other non-belief characteristics of households such as 1) elasticity of consumption to discount rate shock, or elasticity of intertemporal substitution (**EIS**), 2) exposures of their personal income to aggregate output fluctuation (**Exposure**), 3) preferences between current and future consumption, measured by discount factor or marginal propensity to consume (**MPC**). These factors interact with the macroeconomic expectations to influence individual's consumption. This implies that the aggregate exposure to discount factor shocks and, more importantly, the aggregate MPC, a critical sufficient statistics Auclert *et al.* (2018) for shock transmissions, depend on not only the distribution of the non-belief characteristics but also the macroeconomic expectations.

This paper studies the importance of belief heterogeneity of households. First, I analyze how non-belief characteristics correlates with expectations on future one-year ahead unemployment rate changes. I impute an MPC and an Exposure to each individual in Michigan Survey of Consumer using empirical techniques in Patterson (2023) and I use stock market . Then, using the latent variables design from Bhandari *et al.* (2019) and Mankiw *et al.* (2003), I estimate the average forecast of one-year ahead unemployment rate changes for each type of households. I find that households with high EIS or low MPC tend to update their forecasts more rapidly in response to the business cycle fluctuation.

I calibrate a heterogeneous-agent model with incomplete information that produces the empirical pattern of macroeconomic expectations and I show that the model implies more amplification of demand shocks and vastly different consumption paths of each type of households, compared to a model with same non-belief characteristics but without belief heterogeneity.

Literature Das *et al.* (2020) finds that macroeconomic expectations are correlated with the socioeconomic status, such as income and education. My empirical focuses on the correlation that are more relevant in quantitative heterogeneous-agent models. Angeletos and Huo (2021), Angeletos and Lian (2022) and Auclert *et al.* (2020) incorporate information frictions to consumption-saving problems, but heterogeneity in information frictions was yet to be explored in their work. To the best of my knowledge,Guerreiro (2022) is the first to discuss the theoretical potential of macro shock amplification through correlation of beliefs and exposures to business cycle. This paper focuses on the quantitative assessment of the impact of correlation between a wide range of non-belief characteristics (Exposures, MPC and EIS) and macroeconomic expectations.

The paper proceeds as follows. Section 2 discusses empirical findings. Section 3 and 4 describe a heterogeneous-agent model with incomplete information and discuss how information plays a role. Section 8 quantifies importance of heterogeneous beliefs. Section 9 concludes.

2 Beliefs Heterogeneity in Data

In this section, I explore the correlation between non-belief characteristics and macroeconomic expectations. First, I describe the construction of a micro-level dataset of macroeconomic expectations, Exposures, MPCs and EISs. Then, I examine how households with different non-belief characteristics update their beliefs differently in response to business cycle fluctuations.

Following the methodology by Patterson (2023), I estimate the exposure of personal income to business cycle fluctuations for households with different ages, income levels and education attainments using Panel Studies of Income Dynamics (PSID). Exposures are imputed to each household in MSC, based on survey respondents' age, income level and education.

I take the estimate of MPCs for each income group from Patterson (2023) and impute the MPCs using their income. For the EISs, Guvenen (2006) shows that stockholders and non-stockholders respond to an interest rate. I identify high or low EIS using survey respondents' answer of stock market participation.

I focus on the survey question of "Do you think that there will be more unemployment than now, about the same, or less?". Responses of this question take the forms of "more", "less" or "about the same". A challenge for using MSC data to study households' macroeconomic expectation is to transform qualitative forecasts to quantitative forecasts. I extend the method in Bhandari *et al.* (2019) and Mankiw *et al.* (2003). I assume that households from each group has a quantitative forecast around the mean forecast of the group, and they answer "more" or "less" if their quantitative forecast exceeds group-specific upper or lower thresholds.

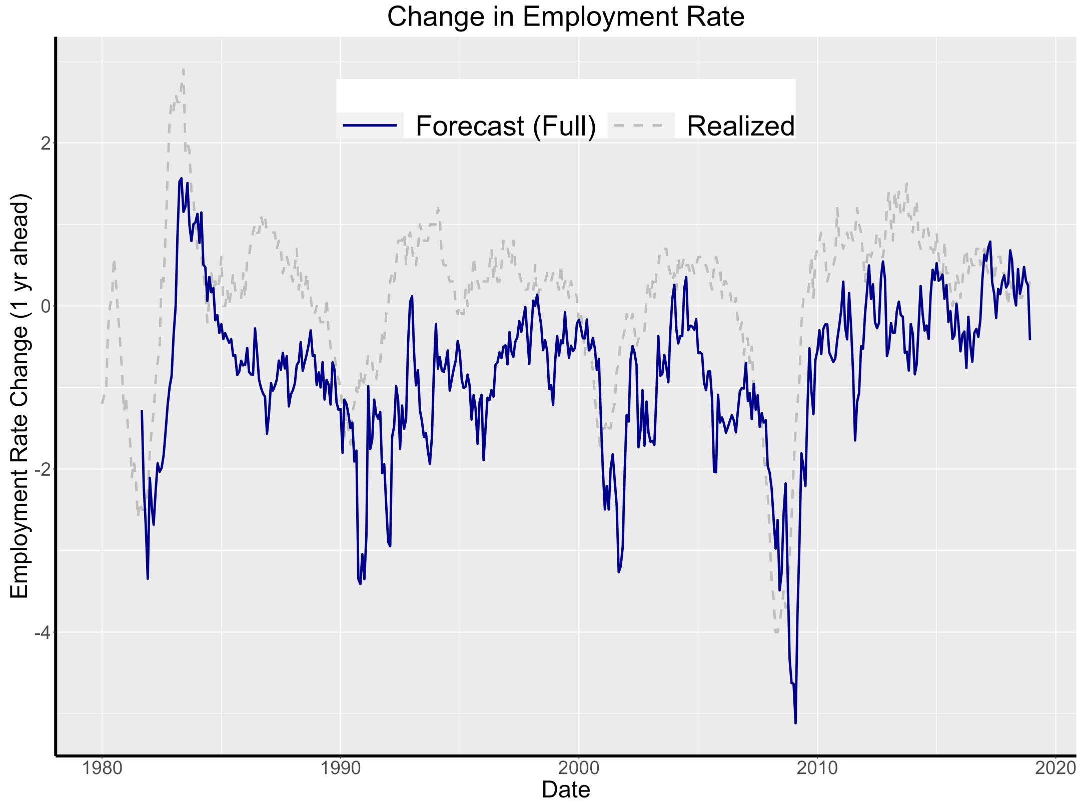


Figure 1: Average forecasted vs realized employment change in a year

Figure 1 shows that the estimated average quantitative forecast of employment changes tracks the realized change in employment rate. Consistent with findings in literature XXX Cite XXX, households' forecast tends to be more pessimistic than realized macroeconomic conditions. To isolate co-movements of the forecasts and the business cycle from general optimism or pessimism, I regress group-average forecasts to realized employment rate changes.

$$\mu_{g,t} = \alpha_{0,g} + \alpha_{1,g}\Delta y_t + \epsilon_{g,t} \quad \text{with} \quad E[\epsilon_{g,t}] = 0 \tag{1}$$

$\alpha_{0,g}$ captures general optimism or pessimism for group g. $\alpha_{1,g}$ captures expectation changes in response to business cycle fluctuations. To understand how two groups (g and g') respond to business cycle fluctuations differently, we can regress differences of macroeconomic expectations between the two groups to realized employment change.

$$\mu_{g,t} - \mu_{g',t} = (\alpha_{0,g} - \alpha_{0,g'}) + (\alpha_{1,g} - \alpha_{1,g'})\Delta y_t + \epsilon_{g,t} - \epsilon_{g',t} \quad \text{with} \quad E[\epsilon_{g,t} - \epsilon_{g',t}] = 0 \tag{2}$$

In the absence of belief heterogeneity, the regression coefficients across groups are identical. Thus, $\alpha_{0,g} - \alpha_{0,g'} = 0$ and $\alpha_{1,g} - \alpha_{1,g'} = 0$. If there is only a difference in general optimism or pessimism between groups, then only $\alpha_{1,g} - \alpha_{1,g'} = 0$. Belief heterogeneity is irrelevant to business cycle fluctuations in this case because it means that all households update their beliefs in the same way when the economy is hit by a shock. Thus, I test $H_0 : \alpha_{1,g} - \alpha_{1,g'} = 0$ to see how households update their beliefs differently.

Table 1: Forecast Differences Between Groups on Realized Employment Change

	Dependent variable:		
	(High - Low) Exposure	(High - Low) MPC	(High - Low) EIS
	(1)	(2)	(3)
Realized	−0.022	−0.049***	0.072***
	(0.019)	(0.015)	(0.015)
Constant	0.254***	−0.536***	0.411***
	(0.028)	(0.023)	(0.027)
Observations	490	490	295
R^2	0.003	0.021	0.073
Adjusted R^2	0.001	0.019	0.070
Residual Std. Error	0.618 (df = 488)	0.504 (df = 488)	0.460 (df = 293)
F Statistic	1.435 (df = 1; 488)	10.265*** (df = 1; 488)	23.233*** (df = 1; 293)

Note: *p<0.1; **p<0.05; ***p<0.01

Table 1 uncovers very different patterns across types of characteristics. The difference of forecasts between high and low Exposures groups is acyclical, even though high Exposures group is more optimistic than the low Exposures group in general. The difference of forecasts between high and low MPCs groups is countercyclical. One possibility is that low MPC households are more forward-looking and thus pay differentially more attention to future economic conditions. They are more accurate in forecasting and thus their $\alpha_{1,g}$ is larger. Finally, the difference of forecasts between high and low EIS group is procyclical. Since EIS is approximated by stock market participation, the high EIS households in the data may pay more attention to the state of economy. Therefore, they update their forecasts more rapidly to changes in economic conditions.

All in all, the empirical finding suggests that belief heterogeneity across MPC types or EIS types seem to be more salient during business cycle fluctuations.

3 Model

In this section, I describe a consumption-saving model with rich heterogeneity in non-belief characteristics (EIS, MPC and Exposure) as well as expectation formations.

3.1 Consumption

There are G types of households, indexed by $g \in \{1, 2, ..., G\}$, with corresponding population of π_g. Each consumer in group g survives with a probability of $\omega_g \in (0, 1]$ and dies with a probability $1 - \omega_g$. Differences in survival probability generate heterogeneity in patience and, therefore, heterogeneity in MPCs. The interest rate is exogenously set at R. To make the model tractable as in Angeletos and Huo (2021), Blanchard (1985) and Yaari (1965), consumers can save in actuarially fair annuities, with return of R/ω_g.

3.1.1 Household Maximization Problem

For a consumer i of type g, born in period τ, the lifetime utility for a given consumption path at birth is given by

$$\sum_{t=\tau}^{\infty} (\chi\omega_g)^{t-\tau} E_t[\Phi_{i,g,t}(C_{i,g,\tau;t})^{1+1/\sigma_g}]$$

$\Phi_{i,t}$ is a shock to discount rate. Each group has a group-specific elasticity of intertemporal substitution to match the empirical heterogeneity in EIS Guvenen (2006). The budget constraint is given by

$$C_{i,g,\tau;t} + S_{i,g,\tau;t} = \frac{R}{\omega_g} S_{i,g,\tau;t-1} + Y_{i,g,t} + T_{g,t}$$

Individual income is given by an idiocyncratic component and an aggregate compoent, $Y_{i,g,t} = \exp(\epsilon^y_{i,t})(Y_t)^{\lambda_g}$. λ_g is the elasticity of individual income to aggregate output, which captures the differential exposures of personal income to business cycle. $\epsilon^y_{i,t}$ is an idiocyncratic shock to income. Households only observe $\exp(\epsilon^y_{i,t})(Y_t)^{\lambda_g}$ as a whole instead of $\epsilon^y_{i,t}$ and Y_t separately.

After linearization around the steady state of $\chi_t R = 1$, optimal consumption (in log derivation from the steady state) is given by

$$\begin{aligned} c_{i,g,\tau;t} =& \frac{1-\chi\omega_g}{\chi\omega_g} s_{i,g,\tau;t-1} - \chi\omega_g\sigma_g\big[\phi_{i,g,t} + \sum_{k=1}^{\infty}(\chi\omega_g)^k E_{i,g,\tau;t}[\phi_{i,g,t+k}]\big] \\ &+ (1-\chi\omega_g)\big[y_{i,t} + \sum_{k=1}^{\infty}(\chi\omega_g)^k E_{i,g,\tau;t}[y_{i,t+k}]\big] \end{aligned}$$

where $\phi_{i,g,t}$ is log deviation of $\Phi_{i,g,t+1}/\Phi_{i,g,t}$. The growth of individual discount factor is assumed to be

$$\phi_{i,g,t} = \phi_t + \epsilon_{i,g,t}$$

where ϕ_t is an aggregate shock to the discount factor. In this model, it plays a role of generating business cycles. The idiocyncratic element $\epsilon_{i,g,t}$ prevents households from observing aggregate shocks directly. This will be discussed later in details in Section 3.3. The log-derivation of personal income $y_{i,t}$ is given by $y_{i,t} = \lambda_g y_t + \epsilon^y_{i,t}$. After aggregation, average optimal consumption of group g (in log derivation from the steady state) is given by

$$c_{g,t} =(1-\beta_g)Rs_{g,t-1} - \beta_g\sigma_g\big[\phi_t + \sum_{k=1}^{\infty}(\beta_g)^k \bar{E}_{g,t}[\phi_{t+k}]\big] + (1-\beta_g)\lambda_g\big[y_t + \sum_{k=1}^{\infty}(\beta_g)^k \bar{E}_{g,t}[y_{t+k}]\big] \tag{3}$$

where β_g is the effective discount rate that takes into account of the survival probability,

$\beta_g = \chi\omega_g$. The group level budget constraint is given by

$$c_{g,t} + s_{g,t} = \frac{1}{\chi}s_{g,t-1} + \lambda_g y_t \tag{4}$$

3.2 Market Clearing Condition

Aggregate output of this economy is demand-determined, which is the sum of the consumption of all groups.

$$y_t = \sum_g \pi_g \Big[(1-\beta_g)Rs_{g,t-1} - \beta_g\sigma_g\big[\phi_t + \sum_{k=1}^{\infty}(\beta_g)^k \bar{E}_{g,t}[\phi_{t+k}]\big] + (1-\beta_g)\lambda_g\big[y_t + \sum_{k=1}^{\infty}(\beta_g)^k \bar{E}_{g,t}[y_{t+k}]\big]\Big] \tag{5}$$

Condition 5 states that aggregate output is affected by demand shocks ϕ_t, beliefs about future demand shocks ϕ_{t+k}, and beliefs about future outputs y_{t+k}. By repeating substitution, aggregate output y_t is given by the first order beliefs as well as the higher-order beliefs on ϕ_t. In Section 4, I will describe how the model can be solved.

3.3 Uncertainty and Expectation Formation

Fundamental Uncertainty: ϕ_t follows an $AR(1)$ process

$$\phi_t = \rho\phi_{t-1} + \eta_t$$

with

$$\eta_t \sim N(0, (\tau^{\phi})^{-1})$$

Households do not observe ϕ_t directly. Instead, they observe their own realization of discount factor shocks, which is given by

$$\phi_{i,g,t+k} = \phi_t + \epsilon_{i,g,t} \tag{6}$$

where

$$\epsilon_{i,g,t} \sim N(0, (\tau_g^x)^{-1})$$

In addition, households observe their personal income $y_{i,t}$, which can be used for forecasting ϕ_t. For now, I assume that the variance of the idiocyncratic shock $\epsilon_{i,t}^y$ to be infinitely large. This avoids the complication of extracting information from endogenous signals.

Lastly, notice that since ϕ_t is unobserved and affects aggregate outputs according to equation (5), this implies that households are also uncertain about aggregate outputs y_t.

Rational Expectation: I assume households form expectations about y_t and ϕ_t using the full history of realized discount rate shocks $\{\phi_{i,t-k}\}_{k=0}^{\infty}$ and the knowledge of the model. Rationality is common knowledge. Thus, each household understands that other households are rational. As discussed before, since households use personal shocks realization $\{\phi_{i,t-k}\}_{k=0}^{\infty}$ to forecast, it implies that each household would have different opinions about the demand shock ϕ_t as well as y_t due to the market clearing condition 5. This is consistent with the data that households are uncertain about both present and future economic conditions.

3.4 Rational Expectation Equilibrium

A Rational Expectation Equilibrium of this model is given by an aggregate output process y_t that satisfies

1. Optimal consumption

$$c_{g,t} = (1-\beta_g)Rs_{g,t-1} - \beta_g\sigma_g\big[\phi_t + \sum_{k=1}^{\infty}(\beta_g)^k\bar{E}_{g,t}[\phi_{t+k}]\big] + (1-\beta_g)\lambda_g\big[y_t + \sum_{k=1}^{\infty}(\beta_g)^k\bar{E}_{g,t}[y_{t+k}]\big] \tag{7}$$

2. Market Clearing

$$y_t = \sum_g \pi_g c_{g,t} \tag{8}$$

3. Expectation is formed with signals and the knowledge of the model. Rationality is common knowledge.

4 Model Solution

Characterizing the solution of an incomplete information model is often challenging. As discussed before, higher-order beliefs of households about demand shocks are involved to compute y_t. To form expectations on these higher-order beliefs, households need to use all signals in the past, which leads to an infinite regress problem.

The solution strategy in this paper is to first form an educated guess that the process y_t follows an $MA(\infty)$ process of the fundamental η_t. Given the coefficients of the process, households only need to form predictions on the path of η_t, which can be done via a standard Kalman smoother.

The coefficients of the $MA(\infty)$ process can be pinned down by a fixed point problem in the impulse response function (IRF). This requires computing the IRF of consumption and therefore, the evolution of the beliefs. But given the coefficients of the process, only evolution of beliefs on η_t needs to be tracked which is also a standard application of the Kalman smoother.

Proposition 1 *The impulse response function for the aggregate output denoted by* $\boldsymbol{h}_y = \begin{bmatrix} \frac{dy_0}{d\eta_0} & \frac{dy_1}{d\eta_0} & \dots \end{bmatrix}'$ *is the solution of the following linear system*

$$\boldsymbol{h}_{c,g} = (1-\beta_g)R\boldsymbol{L}\boldsymbol{h}_{s,g} - \beta_g\sigma_g(\boldsymbol{h}_\phi + \boldsymbol{W}_g\boldsymbol{h}_\phi) + (1-\beta_g)\lambda_g(\boldsymbol{h}_y + \boldsymbol{W}_g\boldsymbol{h}_y) \tag{9}$$

$$\boldsymbol{h}_{c,g} + \boldsymbol{h}_{s,g} = R\boldsymbol{L}\boldsymbol{h}_{s,g} + \lambda_g\boldsymbol{h}_y \tag{10}$$

$$\boldsymbol{h}_y = \sum_{g=1}^{G} \pi_g \boldsymbol{h}_{c,g} \tag{11}$$

where $\boldsymbol{h}_{c,g} = \begin{bmatrix} \frac{dc_{g,0}}{d\eta_0} & \frac{dc_{g,1}}{d\eta_0} & \dots \end{bmatrix}'$ *and* $\boldsymbol{h}_{s,g} = \begin{bmatrix} \frac{dc_{g,0}}{d\eta_0} & \frac{dc_{g,1}}{d\eta_0} & \dots \end{bmatrix}$ *are the IRFs for consumption and saving of group g respectively.* $\boldsymbol{L} = \begin{bmatrix} \boldsymbol{0}_{\infty\times 1} & \boldsymbol{I} \end{bmatrix}'$ *is the lag operator in the matrix form and* $\boldsymbol{W}_g$ *is* *the expectation matrix for* η_t*, where*

$$\boldsymbol{W}_g = \begin{bmatrix} m_g' \boldsymbol{M}_{g,0}^{\eta} \\ m_g' \boldsymbol{M}_{g,1}^{\eta} \\ \vdots \end{bmatrix}$$

with $m_g' = \begin{bmatrix} \beta_g & \beta_g^2 & \beta_g^3 & \dots \end{bmatrix}$ *and* $\boldsymbol{M}_{g,t}^{\eta} \equiv \begin{bmatrix} 0 & \frac{d\bar{E}_{g,t}[\eta_t]}{d\eta_0} & \frac{d\bar{E}_{g,t}[\eta_{t-1}]}{d\eta_0} & \dots \\ 0 & 0 & \frac{d\bar{E}_{g,t}[\eta_t]}{d\eta_s} & \dots \\ \vdots & \vdots & \vdots & \dots \end{bmatrix}$

The equation (9) comes from taking derivative of the optimal consumption. The $\boldsymbol{W}_g$ matrix tracks the update of the beliefs on η_t with proper weighting of the discount factors. The derivation is detailed in appendix B. The equation (11) is the market clearing condition in all periods and the equation (10) is the budget constraint.

Connection to Higher-Order Belief: The equations (9), (10), and (11) pin down the IRF for the aggregate output

$$\boldsymbol{h}_y = \boldsymbol{M}_\phi \boldsymbol{h}_\phi + \boldsymbol{M}_y \boldsymbol{h}_y \tag{12}$$

The exact formula for $\boldsymbol{M}_\phi$ and $\boldsymbol{M}_y$ are in the appendix B. Via repeated substitution, the IRF of the aggregate output is given by the IRFs of the ϕ

$$\boldsymbol{h}_y = \boldsymbol{M}_\phi \boldsymbol{h}_\phi + \boldsymbol{M}_y \boldsymbol{M}_\phi \boldsymbol{h}_\phi + \boldsymbol{M}_y^2 \boldsymbol{M}_\phi \boldsymbol{h}_\phi + \dots$$

This is analogous to the result in Beauty Contest that the equilibrium quantity is often the sum of higher-order beliefs (Angeletos and Lian (2022), Angeletos and Lian (2018), and Angeletos and Huo (2021)). The aggregate output can be neatly solved by inverting a matrix

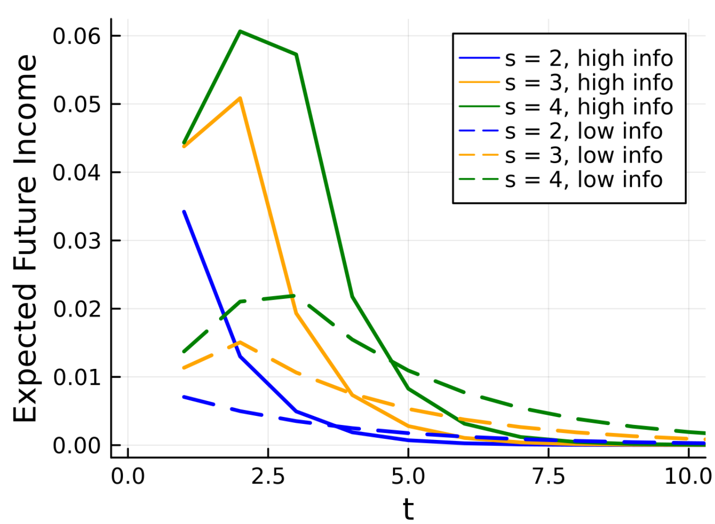
$$\boldsymbol{h}_y = (\boldsymbol{I} - \boldsymbol{M}_y)^{-1} \boldsymbol{M}_\phi \boldsymbol{h}_\phi$$

Connection to Intertemporal MPC: In case of no imperfect information, it can be shown that matrix $\boldsymbol{M}_y$ reduces to a standard iMPC matrix in Auclert *et al.* (2018). The joint distribution of signal precisions, Exposures and MPCs determines the elements of $\boldsymbol{M}_y$.

5 Role of Information Allocation

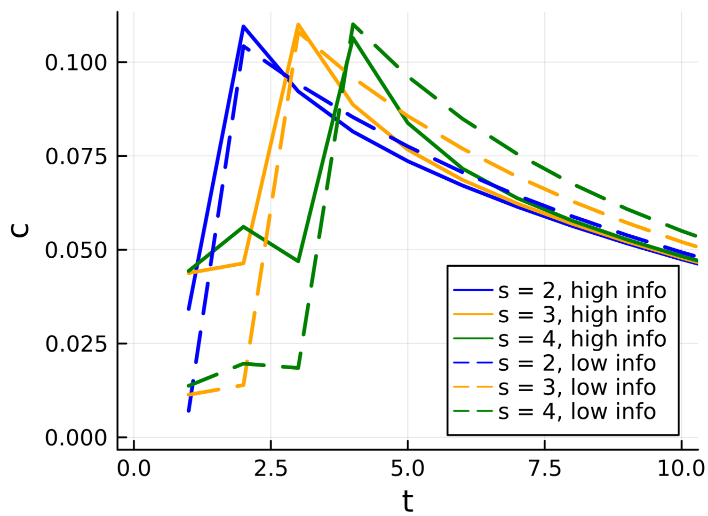
How does information interact with other household characteristics? In this model, information affects the expectation of future variables and thus the consumption response. For an arbitrary process $y_t = \eta_{t-s}$, the IRF of $\sum_{k=1}^{\infty}(\beta_g)^k \bar{E}_{g,t}[y_{t+k}]$ is given by the s column of the matrix $\boldsymbol{W}_g$

Figure 2: IRF of $\sum_{k=1}^{\infty}(\beta_g)^k \bar{E}_{g,t}[y_{t+k}]$



Before time t, households with high-quality signal responds more while the households with low-quality has a muted response. This is because as η_0 is shocked, the households with high-quality signal are able to update their beliefs on η_0 and y_t quickly. Since households are forward-looking, households with high-quality signal react vigorously in anticipation for the future increase in y_t. As time goes by, households are collecting more information about the sequence of η_t. The households with high-quality signal are able to update their beliefs on η_t for $t > 0$ and the shock on η_0 ceases to have an effect. On the other hand, the households with low-quality do not have precise information on the future η_t, so the shock on η_0 makes a long-lasting effect on their beliefs on η_t for $t > 0$. This explains why the IRF for the households with low-quality signal is more persistent even after time t.

Figure 3: IRF of consumption caused by expectation



This pattern is similar when saving is considered. In figure (3), I plotted the IRFs of consumption response to an income process of $y_t = \eta_{t-s}$. This is the columns of $(1 - \beta_g)\boldsymbol{B}_g(\boldsymbol{A} + \boldsymbol{I} + \boldsymbol{W}_g)$ matrix. The information shifts the intertemporal MPC from future to present. As saving is considered, the households with high-quality signal are responding less at the realization of the income shock because they have already accumulated debts from the earlier consumption. As they start to repaying debt, their consumption response is slightly muted compared to the households with low-quality information.

To decompose the effect of information heterogeneity, a decomposition is performed.

Recall that the consumption response when saving is considered is given by

$$\boldsymbol{h}_{c,g} = (1-\beta_g)R\boldsymbol{L}\boldsymbol{h}_{s,g} - \beta_g\sigma_g(\boldsymbol{h}_\phi + \boldsymbol{W}_g\boldsymbol{h}_\phi) + (1-\beta_g)\lambda_g(\boldsymbol{h}_y + \boldsymbol{W}_g\boldsymbol{h}_y) \tag{13}$$

$$\boldsymbol{h}_{c,g} + \boldsymbol{h}_{s,g} = R\boldsymbol{L}\boldsymbol{h}_{s,g} + \lambda_g\boldsymbol{h}_y \tag{14}$$

$$\implies \boldsymbol{h}_{c,g} = \underbrace{-\beta_g\sigma_g\boldsymbol{B}_g(\boldsymbol{h}_\phi + \boldsymbol{W}_g\boldsymbol{h}_\phi)}_{\text{Direct Effect}} + \underbrace{(1-\beta_g)\boldsymbol{B}_g\lambda_g(\boldsymbol{A}\boldsymbol{h}_y + \boldsymbol{h}_y + \boldsymbol{W}_g\boldsymbol{h}_y)}_{\text{Indirect Effect}} \tag{15}$$

with $\boldsymbol{A} = R\boldsymbol{L}(\boldsymbol{I} - R\boldsymbol{L})^{-1}$ and $\boldsymbol{B}_g = (\boldsymbol{I} + (1-\beta_g)\boldsymbol{A})^{-1}$. This allows us to distinguish the direct and indirect effect of the shock to consumption. The direct effect is given by the realized shocks and the expectation of the shocks while the indirect effect is given by the feedback of changes in income.

Since W_g is the only term that involves households' information set, this allows an intuitive decomposition to isolate the effect of information heterogeneity. The IRF of consumption of each group can be decomposed as four components - direct and indirect effect in a model without information heterogeneity and the additional direct and indirect effect due to information heterogeneity.

$$\boldsymbol{h}_{c,g} = \boldsymbol{h}_{c,g}^{PE,uncor} + \boldsymbol{h}_{c,g}^{GE,uncor} + (\boldsymbol{h}_{c,g}^{PE} - \boldsymbol{h}_{c,g}^{PE,uncor}) + (\boldsymbol{h}_{c,g}^{GE} - \boldsymbol{h}_{c,g}^{GE,uncor}) \tag{16}$$

$\boldsymbol{h}_{c,g}^{PE,uncor}$ and $\boldsymbol{h}_{c,g}^{GE,uncor}$ are the PE and GE components of the impulse response function of consumption in an incomplete information HANK model. They are given by

$$\boldsymbol{h}_{c,g}^{PE,uncor} = -\beta_g\sigma_g\boldsymbol{B}_g(\boldsymbol{h}_\phi + \bar{\boldsymbol{W}}_g\boldsymbol{h}_\phi) \tag{17}$$

$$\boldsymbol{h}_{c,g}^{GE,uncor} = \boldsymbol{B}_g(1-\beta_g)\lambda_g(\boldsymbol{A}\boldsymbol{h}_y + \boldsymbol{h}_y + \bar{\boldsymbol{W}}_g\boldsymbol{h}_y) \tag{18}$$

where

$$\bar{\boldsymbol{W}}_g = \begin{bmatrix} m_g' \boldsymbol{M}_0^\eta \\ m_g' \boldsymbol{M}_1^\eta \\ \vdots \end{bmatrix}$$

with $m_g' = \begin{bmatrix} \beta_g & \beta_g^2 & \beta_g^3 & \dots \end{bmatrix}$ and $\boldsymbol{M}_t^\eta \equiv \begin{bmatrix} 0 & \frac{d\bar{E}_t[\eta_t]}{d\eta_0} & \frac{d\bar{E}_t[\eta_{t-1}]}{d\eta_0} & \dots \\ 0 & 0 & \frac{d\bar{E}_t[\eta_t]}{d\eta_s} & \dots \\ \vdots & \vdots & \vdots & \dots \end{bmatrix}$

$\bar{E}_t[.]$ is the average expectation across the economy. For an economy with an equal size of high-quality and low-quality signal households, if $\bar{E}_t^h[.]$ is the expectation of the high-quality signal households while $\bar{E}_t^l[.]$ is the expectation of the low-quality signal households, then $\bar{E}_t[.] = \frac{1}{2}(\bar{E}_t^h[.] + \bar{E}_t^l[.])$.

The last two terms $\boldsymbol{h}_{c,g}^{PE} - \boldsymbol{h}_{c,g}^{PE,uncor}$ and $\boldsymbol{h}_{c,g}^{GE} - \boldsymbol{h}_{c,g}^{GE,uncor}$ measure the additional consumption response due to information heterogeneity. They are given by

$$\boldsymbol{h}_{c,g}^{PE} - \boldsymbol{h}_{c,g}^{PE,uncor} = -\beta_g \sigma_g \boldsymbol{B}_g (\boldsymbol{W}_g - \bar{\boldsymbol{W}}_g) \boldsymbol{h}_\phi \tag{19}$$

$$\boldsymbol{h}_{c,g}^{GE} - \boldsymbol{h}_{c,g}^{GE,uncor} = (1 - \beta_g) \boldsymbol{B}_g \lambda_g (\boldsymbol{W}_g - \bar{\boldsymbol{W}}_g) \boldsymbol{h}_y \tag{20}$$

Similarly, the aggregate consumption response can be decomposed into four components

$$\boldsymbol{h}_c = \boldsymbol{h}_c^{PE,uncor} + \boldsymbol{h}_c^{GE,uncor} + (\boldsymbol{h}_c^{PE} - \boldsymbol{h}_c^{PE,uncor}) + (\boldsymbol{h}_c^{GE} - \boldsymbol{h}_c^{GE,uncor}) \tag{21}$$

where $\boldsymbol{h}_c^{PE,uncor}$ and $\boldsymbol{h}_c^{GE,uncor}$ are population average of $\boldsymbol{h}_{c,g}^{PE,uncor}$ and $\boldsymbol{h}_{c,g}^{GE,uncor}$ respectively.

5.1 Exposure and Information

Now we examine the effect of information to consumption under different Exposure (λ_g). In figure (3), I plotted the IRFs of consumption response to an income process of $y_t = \eta_{t-s}$

with different exposures to business cycle. λ_g scales the consumption response because it determines the change in personal income along the business cycle fluctuation. As households always consume a fraction of their life-time expected income, the consumption response is scaled by λ_g.

Figure 4: Decomposition of Indirect Effect for high λ households

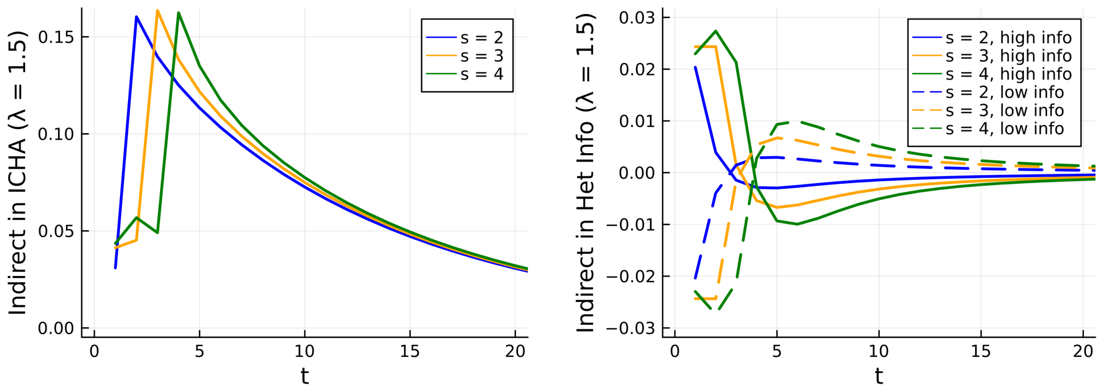
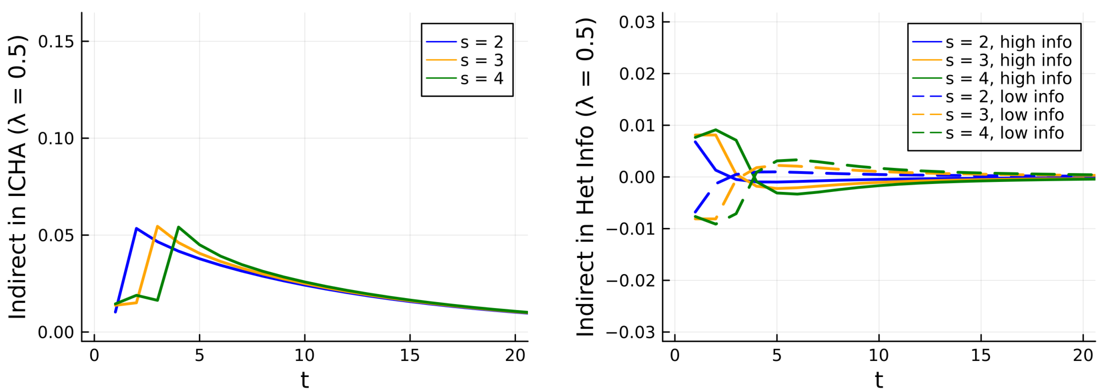
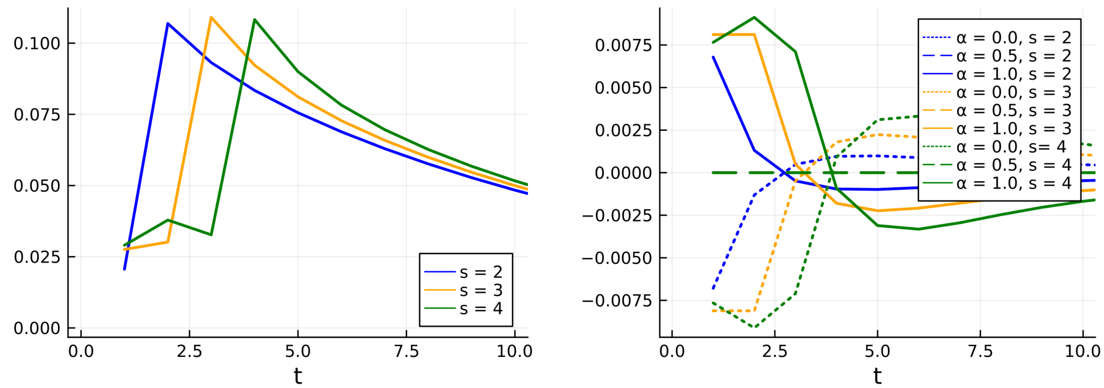


Figure 5: Decomposition of Indirect Effect for low λ households



To understand the importance of heterogeneity and information allocation in aggregate, I consider an economy with two types of households - low exposure and high exposure. Out of the high-exposure households, α of them have high-quality signals and $1-\alpha$ of them have the low-quality signals. The low-exposure households is set in the opposite manner - α of them have low-quality signals and $1-\alpha$ of them have the high-quality signals. In figure (**??**), I compare the aggregate consumption response between an economy where high-exposure households receive more precise signals ($\alpha = 1$) economies and an economy with opposite information allocation ($\alpha = 0$).

Figure 6: Decomposition of Indirect Effect for economies with different α



5.2 MPC and Information

5.2.1 With Saving

Figure 7: Decomposition of Indirect Effect for $\beta = 0.9$

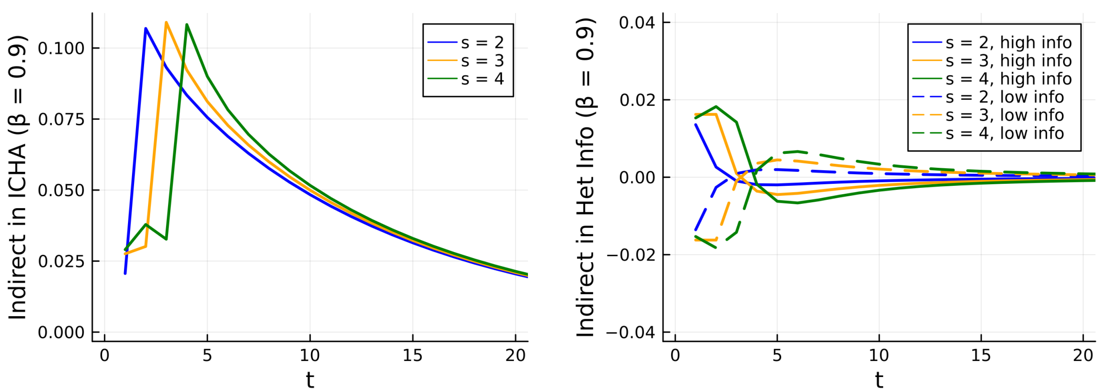


Figure 8: Decomposition of Indirect Effect for $\beta = 0.5$

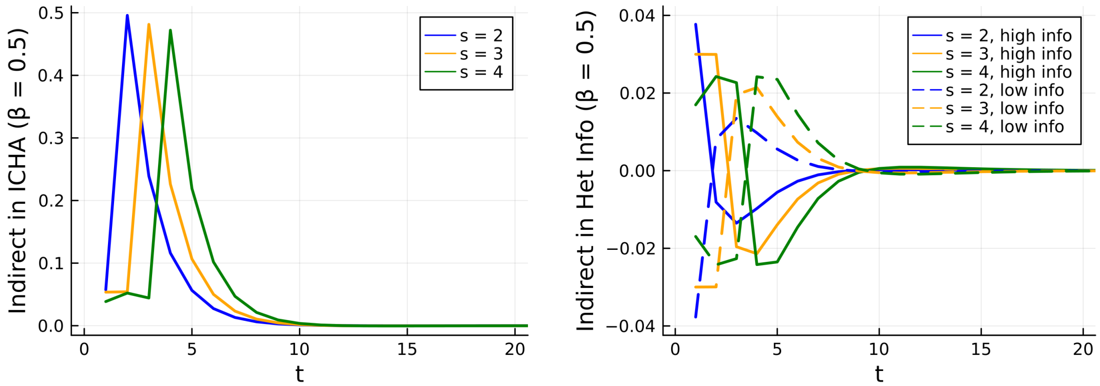


Figure 9: Decomposition of Indirect Effect for $\beta = 0.1$

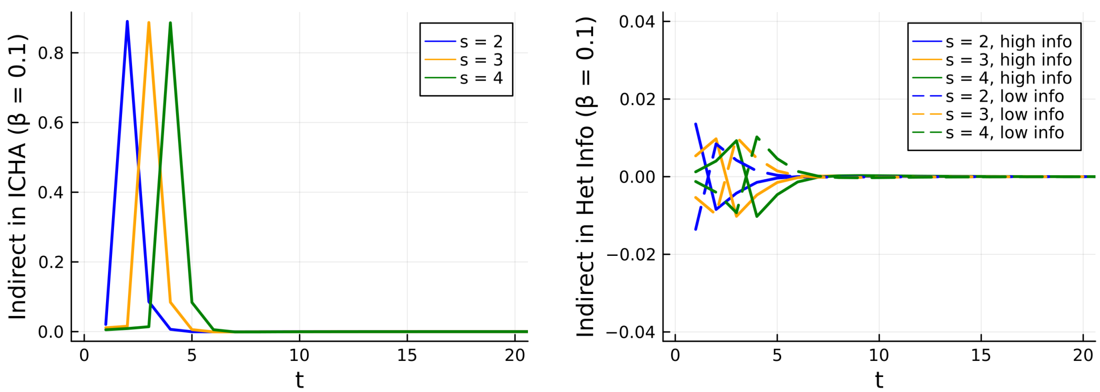
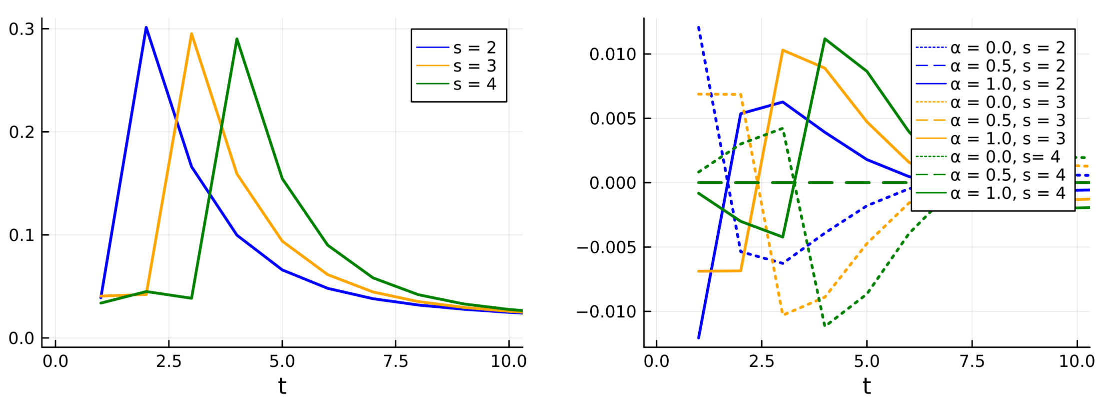


Figure 10: Decomposition of Indirect Effect for $\beta = 0.9$ and $\beta = 0.5$ mix



5.2.2 Without Saving

Figure 11: Decomposition of Indirect Effect for $\beta = 0.9$

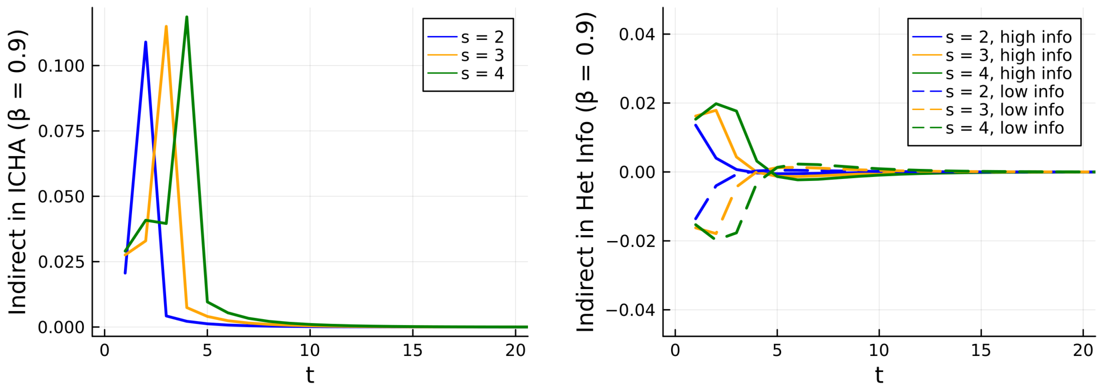


Figure 12: Decomposition of Indirect Effect for $\beta = 0.5$

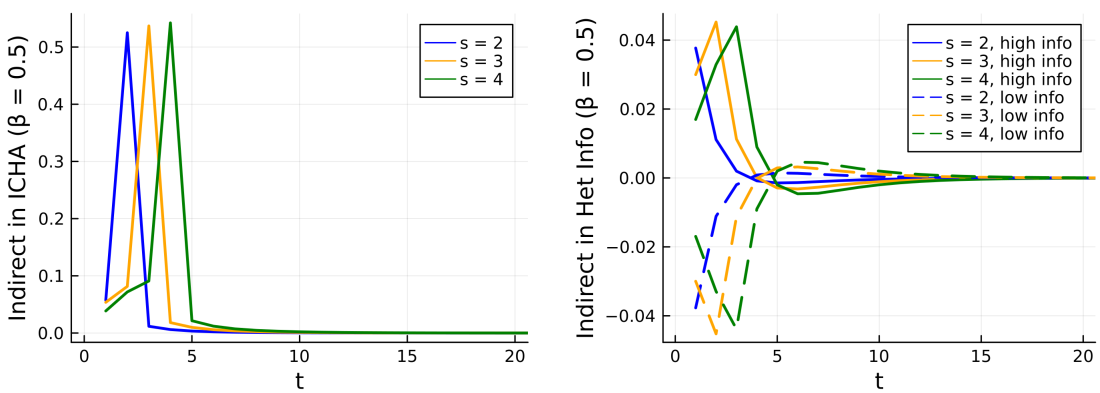


Figure 13: Decomposition of Indirect Effect for $\beta = 0.1$

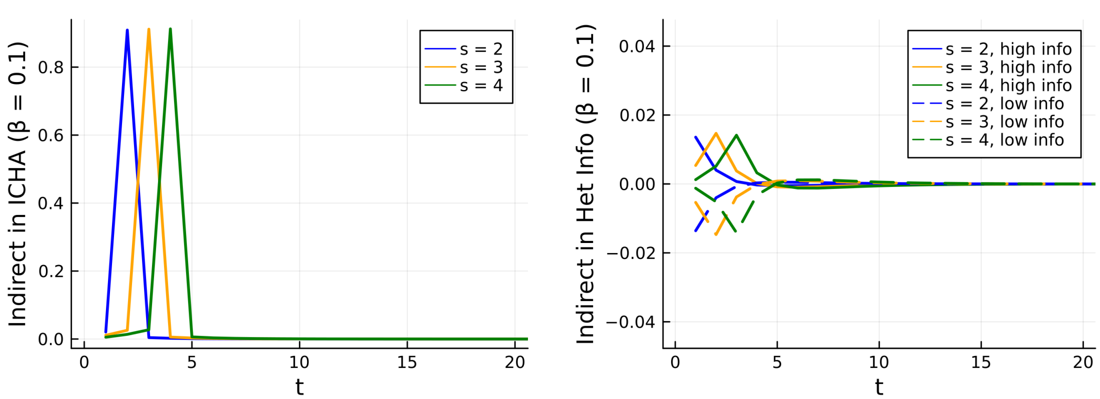
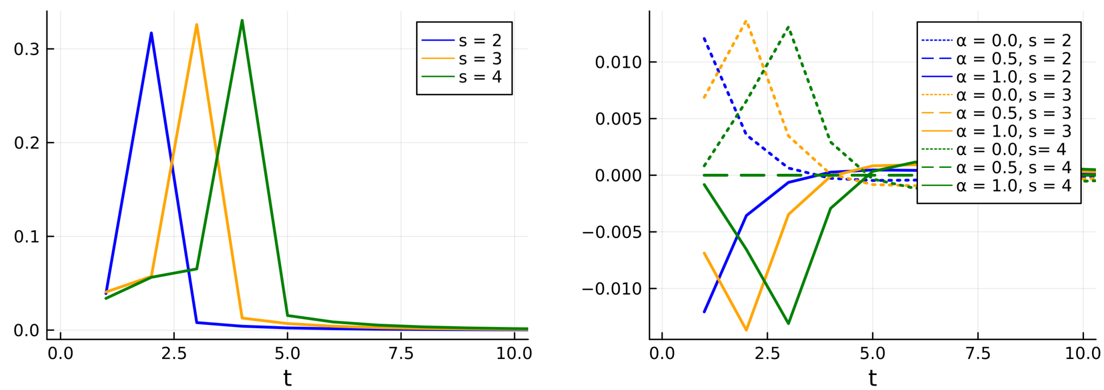


Figure 14: Decomposition of Indirect Effect for $\beta = 0.9$ and $\beta = 0.5$ mix



5.3 EIS and Information

EIS affects the exposure to the discount factor shock. Households with high EIS responds to present and future discount rate shocks more vigorously. Thus, by allocating more information to the high EIS households, the direct consumption response is higher.

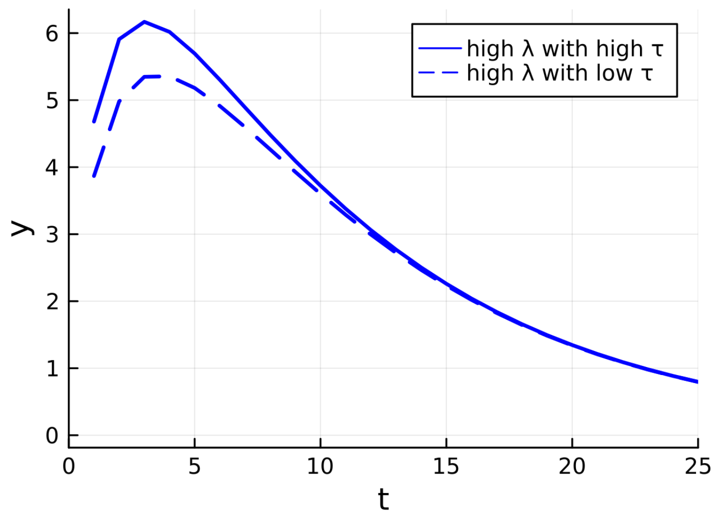
6 Role of Information Allocation (Old, to be removed)

How does information interact with other household characteristics? I compare the IRFs for a discount factor shock ϕ_t under two economies with different information allocations. I set up an economy consisting of two groups of households with distinct non-belief characteristics and distinct signal precisions. Then, I compare it with another economy with same distribution of non-belief characteristics but opposite allocation of signal precisions. When high Exposure or low MPC or high EIS households receive higher quality signals, the output response is amplified.

6.1 Exposure and Information

Households' consumption depends on their future expected personal income. When the aggregate output increases, if high Exposure households have better information, they are more aware of future high increase of personal income. Alternatively, if the low Exposure households have better information, they are more aware of future increase of personal income but the increase in their expected life-time personal income is smaller due to the low Exposure. Therefore, allocating more information to high Exposure households causes bigger increases in consumption, which effectively increases the aggregate MPC of this economy.

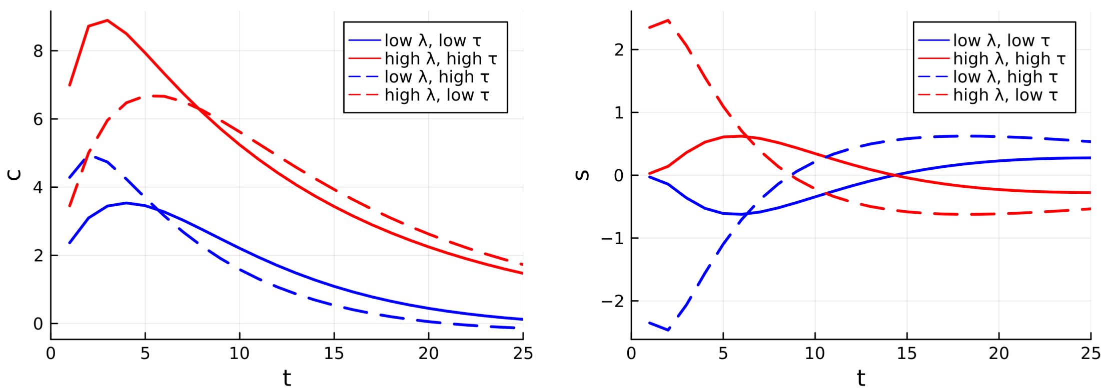
Figure 15: Effect of information allocation to output under heterogeneity in cyclicality



As shown in Figure 15, the economy with high exposure households receiving more precise signals has a higher output response. This is the channel highlighted by Guerreiro (2022).[1] From Figure 16, when there is a positive demand shock, in the economy of high Exposure having better information (solid line), high Exposure households increase consumption massively, compared to high Exposure households in the alternative economy.

[1]Guerreiro (2022) uses cognitive discounting instead of dynamic noisy signals. Both of them dampen the GE response.

Figure 16: Effect of information allocation to consumption and saving under heterogeneity in cyclicality



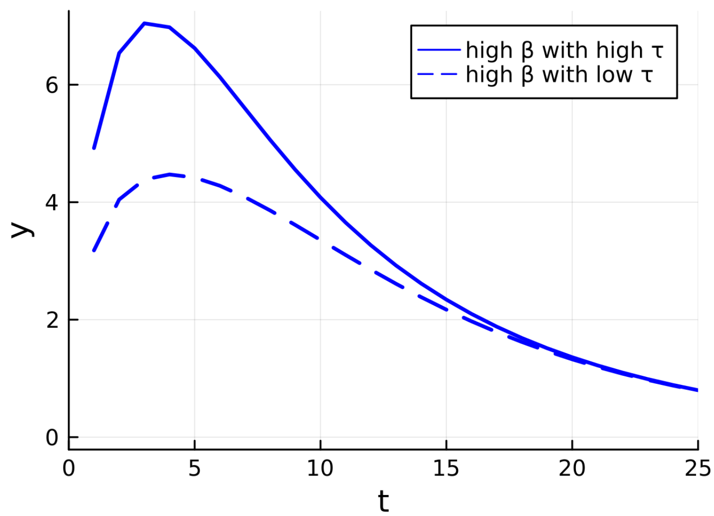
6.2 MPC and Information

MPC affects how forward-looking a household is in a consumption-saving decision. The higher the MPC, the more the consumption depends on expectations of future variables. The effect of information comes in two ways. First, when there is a positive discount factor shock, if low MPC (high discount factor) households have better information, their expectation about future discount factor shock adjusts more rapidly and their consumption response is larger because they put more weights on future variables. If high MPC (low discount factor) households have better information, their expectation adjusts quickly, but their consumption response is limited.

Second, in case of an increase in aggregate output, if low MPC (high discount factor) households have better information, their expectation about future aggregate income increases rapidly and their consumption response is more persistent. Instead, if high MPC (low discount factor) households have better information, their expectation about future aggregate income increases rapidly but their consumption response concentrates on the short run. The effective aggregate MPC can go to both directions.

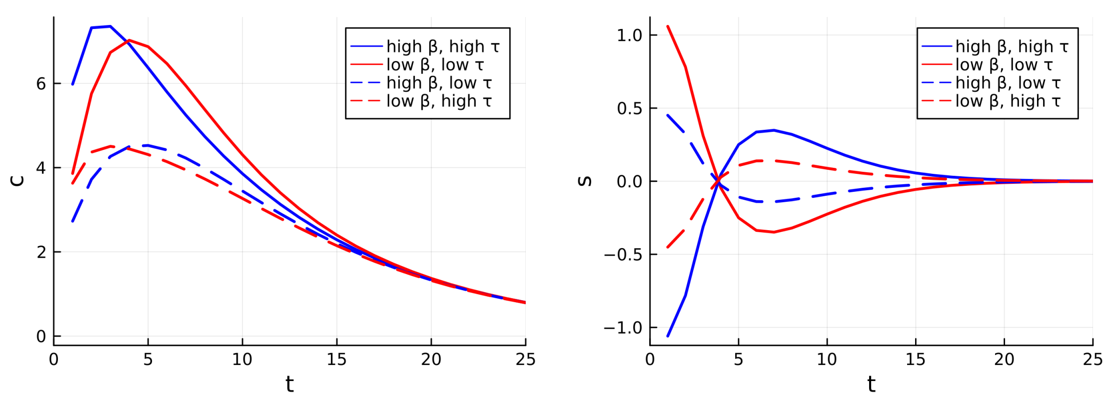
In other words, allocating more information between households with different MPCs has a definite effect on the direct exposure to demand shock M_r but a ambiguous effect on the effective MPC matrix M_y

Figure 17: Effect of information allocation to output under heterogeneity in MPC



As shown in Figure 15, the economy with low MPC households receiving more precise signals produce a higher output response. In Figure 18, the allocation of information has a huge impact to the consumption of each group.

Figure 18: Effect of information allocation to consumption and saving under heterogeneity in MPC



6.3 EIS and Information

EIS affects how susceptible a household is to a shock to discount rate. When there is a positive discount factor shock, if high EIS households have better information, their expectation about the future discount factors adjust more rapidly and their consumption response is bigger since they are more susceptible to shocks to discount rate. Relative to the other two channels, this operates only through M_r.

Figure 19: Effect of information allocation to output under heterogeneity in EIS

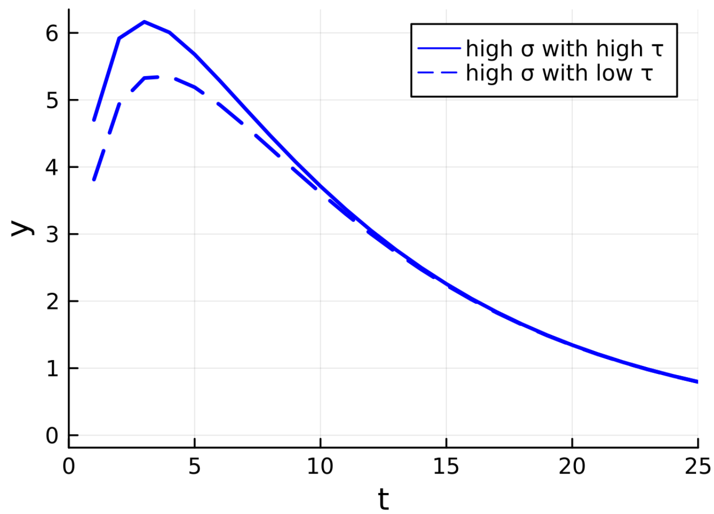
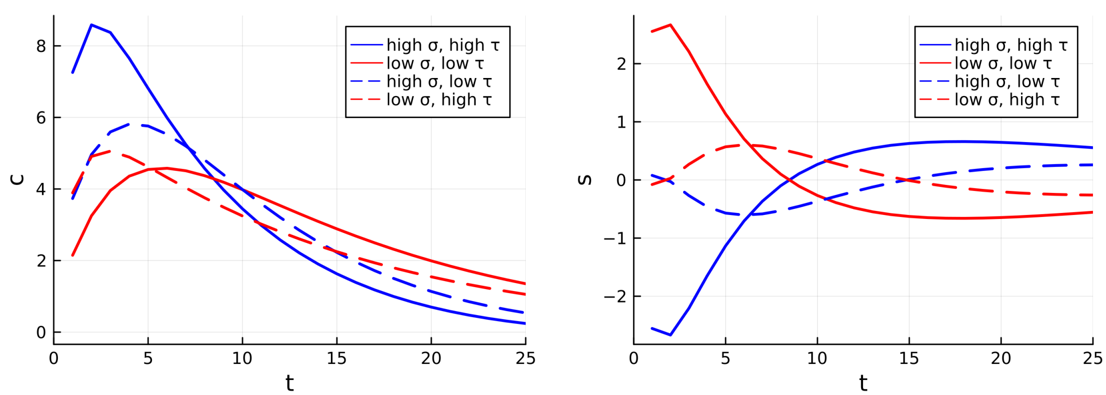


Figure 20: Effect of information allocation to consumption and saving under heterogeneity in EIS



7 Comparative Statics

How does information interact with other household characteristics? We can compute the derivative against an information allocation parameter α.

Recall that

$$\boldsymbol{h}_y = (\boldsymbol{I} - \boldsymbol{M}_y)^{-1}\boldsymbol{M}_\phi \boldsymbol{h}_\phi$$

Proposition 2 *The information allocation effect on* $\boldsymbol{M}_y$ *is given by*

$$\frac{\partial \boldsymbol{h}_y}{\partial \alpha} = (\boldsymbol{I} - \boldsymbol{M}_y)^{-1}\left(\frac{\partial \boldsymbol{M}_y}{\partial \alpha}\boldsymbol{h}_y + \frac{\partial \boldsymbol{M}_\phi}{\partial \alpha}\boldsymbol{h}_\phi\right) \tag{22}$$

The first derivative $\frac{\partial \boldsymbol{M}_y}{\partial \alpha}$ *is the effect of information to the aggregate intertemporal MPC, which is given by*

$$\frac{\partial \boldsymbol{M}_y}{\partial \alpha} = \sum_g \pi_g (1 - \beta_g)\lambda_g \boldsymbol{B}_g \frac{\partial \boldsymbol{W}_g}{\partial \alpha} \tag{23}$$

The second derivative $\frac{\partial \boldsymbol{M}_\phi}{\partial \alpha}$ *is the effect of information to the aggregate direct impact of the shock*

$$\frac{\partial \boldsymbol{M}_\phi}{\partial \alpha} = -\sum_g \pi_g \sigma_g \beta_g \boldsymbol{B}_g \frac{\partial \boldsymbol{W}_g}{\partial \alpha} \tag{24}$$

Obviously, when there is perfect foresight, $\boldsymbol{W}_g$ is independent of information. Thus $\frac{\partial \boldsymbol{W}_g}{\partial \alpha} = 0$.

Two Types only: Suppose there are only two types. Each has a population of $1/2$. For type 1 agents, α of them has high-quality information with precision τ_x^h and $1-\alpha$ of them has low-quality information with precision τ_x^l. The opposite is set for the type 2 agents, $1-\alpha$ of them has high-quality information while the rest has low-quality information.

By increasing α, it is equivalent to transferring some information from type 2 to type 1. Recall that the signal precision affects the equilibrium output through $\bar{E}_{g,t}[\eta_t]$ only, which only depends on whether the group of household receive a high or low quality information. Denote $\boldsymbol{M}_t^{\eta,h}$ and $\boldsymbol{M}_t^{\eta,l}$ as the matrices for high and low precision signals respectively. The $\boldsymbol{M}_{g,t}^{\eta}$ matrix for type 1 and 2 are given by

$$\boldsymbol{M}_{1,t}^{\eta} = \alpha \boldsymbol{M}_{h,t}^{\eta} + (1-\alpha)\boldsymbol{M}_{l,t}^{\eta} \tag{25}$$

$$\boldsymbol{M}_{2,t}^{\eta} = (1-\alpha)\boldsymbol{M}_{h,t}^{\eta} + \alpha \boldsymbol{M}_{l,t}^{\eta} \tag{26}$$

Thus

$$\frac{\partial \boldsymbol{M}_{1,t}^{\eta}}{\partial \alpha} = \boldsymbol{M}_t^{\eta,h} - \boldsymbol{M}_t^{\eta,l}$$
$$\frac{\partial \boldsymbol{M}_{2,t}^{\eta}}{\partial \alpha} = \boldsymbol{M}_t^{\eta,l} - \boldsymbol{M}_t^{\eta,h}$$

λ **only:** Since the β for each group is the same, $m_1' = m_2' = m'$. The effect on the expectation matrix is given by

$$\frac{\partial \boldsymbol{W}_1}{\partial \alpha} = \boldsymbol{W}_h - \boldsymbol{W}_l$$
$$\frac{\partial \boldsymbol{W}_2}{\partial \alpha} = -(\boldsymbol{W}_h - \boldsymbol{W}_l)$$

Therefore

$$\begin{aligned}\frac{\partial \boldsymbol{M}_y}{\partial \alpha} &= \frac{1}{2}(1-\beta)\lambda_1 \boldsymbol{B}(\boldsymbol{W}_h - \boldsymbol{W}_l) - \frac{1}{2}(1-\beta)\lambda_2 \boldsymbol{B}(\boldsymbol{W}_h - \boldsymbol{W}_l) \\ &= \frac{1}{2}(1-\beta)(\lambda_1 - \lambda_2)\boldsymbol{B}(\boldsymbol{W}_h - \boldsymbol{W}_l)\end{aligned}$$

and

$$\frac{\partial \boldsymbol{M}_\phi}{\partial \alpha} = -\frac{1}{2}\beta \boldsymbol{B}(\boldsymbol{W}_h - \boldsymbol{W}_l) + \frac{1}{2}\beta \boldsymbol{B}(\boldsymbol{W}_h - \boldsymbol{W}_l) = 0$$

Thus,

$$\frac{\partial \boldsymbol{h}_y}{\partial \alpha} = \frac{1}{2}(1-\beta)(\lambda_1 - \lambda_2)(\boldsymbol{I} - \boldsymbol{M}_y)^{-1}\boldsymbol{B}(\boldsymbol{W}_h - \boldsymbol{W}_l)\boldsymbol{h}_y$$

To sign these matrices, we need to understand the structure of $\boldsymbol{B}$ and $\boldsymbol{W}_h - \boldsymbol{W}_l$

8 Quantitative Exercise

This section investigates the importance of belief heterogeneity through a fully calibrated model. I divide the population into eight groups. Each group is characterized by high or low MPC, EIS and Exposure. I estimate the correlation of each group's forecast to business cycle fluctuations. Table 2 shows the parameters for the eight groups.

	σ_g	β_g	λ_g	$\alpha_{1,g}$	π_g
1	1.00	0.49	2.32	0.29	0.02
2	1.00	0.49	0.69	0.19	0.20
3	0.10	0.49	2.32	0.12	0.03
4	0.10	0.49	0.69	0.12	0.28
5	1.00	0.95	2.32	0.18	0.12
6	1.00	0.95	0.69	0.23	0.28
7	0.10	0.95	2.32	0.09	0.02
8	0.10	0.95	0.69	0.13	0.05

Table 2: Calibration of the Eight Groups

The group with high EIS, high MPC and high exposure (Group 1) changes their forecast most rapidly in response to the business cycle fluctuations. However, it is one of the smallest group whose impact to the economy is limited. The group with low EIS, high MPC and low exposure (Group 4) and the group with high EIS, low MPC and low exposure (Group 6) highlight the importance of belief heterogeneity. Each of them represents over a quarter of population and their beliefs fluctuates differently along the business cycle.

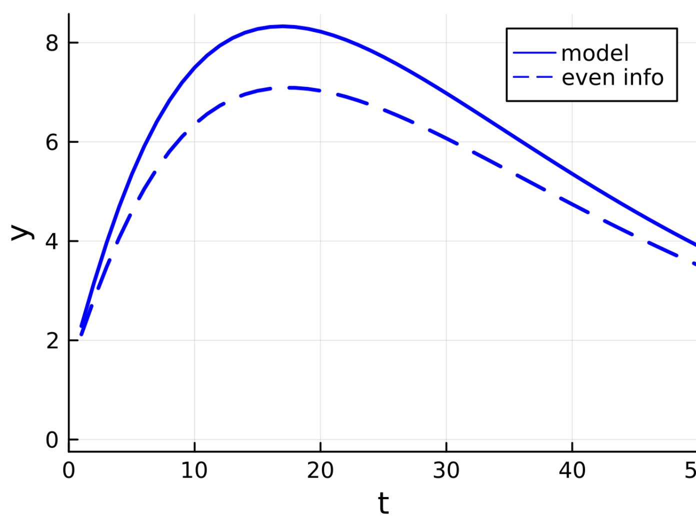
I calibrate the signal precision τ_g^x for each group to match the empirical regression coefficient between group forecast on unemployment changes and realized unemployment changes. This requires computing the analogous regression coefficient in the model. The computation of such coefficient can be performed efficiently, which is documented in Appendix B.5 in detail. $\rho = 0.96$ and $\tau_\eta = 1/4$ are picked to match the time-series properties of the unemployment rate from 1980 to 2019. The calibrated τ_g^x is listed below.

$$\begin{bmatrix} 0.0032 & 0.0013 & 0.00067 & 0.00065 & 0.0012 & 0.0019 & 0.00044 & 0.00071 \end{bmatrix}$$

8.1 Effect of Belief Heterogeneity

How important is belief heterogeneity? In this section, I compare the impulse response function of a demand shock in the calibrated model with an equal information model that has no belief heterogeneity. To construct this equal information model, I reallocate the information set such that for every combination of MPC, EIS and Exposure, there are eight subgroups with signal precision and shares matched to the overall population. This effectively creates $8 \times 8 = 64$ groups and removes the correlation between non-belief characteristics (MPC, EIS and Exposure) and signal quality.

Figure 21: Effect of information allocation to output



As shown in Figure 21, the calibrated economy with belief heterogeneity significantly amplifies the demand shock compared to the economy with equal distribution of information. Figure 22 shows that all eight types of households have higher consumption than the average counterpart in the equal information economy. To understand the mechanism, I decompose the effect of the shock into a direct channel through the demand shock

(PE effect) and an indirect channel through the feedback of income (GE effect).

First, the direct effect is more pronounced in the calibrated economy. This is because high EIS and low MPC households are more informed about the economy in the data. In the equal information economy, when more information is reallocated from high EIS and low MPC households (Group 6) to low EIS and high MPC group (Group 4), the former responds less to the shock while the latter responds roughly the same. Even though Group 7 and Group 8 respond more under the equal information economy in Figure 23, their total share is too small. This leads to an overall increase in PE effect.

Second, the GE effect is also bigger in the calibrated economy. Figure 24 shows that all groups have higher consumption response through the GE effect. To understand the mechanism better, I calculate the GE effects under the same path of output.

In Figure 25, information has very little effect to the high MPC groups, because their consumption mostly follows the current output. The only exception is households with low MPC and high EIS (group 6). When they are more informed, they shifted their spending earlier, which pushes up the aggregate output in the earlier periods. This eventually feeds back to high MPC groups. All in all, the belief heterogeneity has a significant impact to transmission of the demand shock.

9 Conclusion

To conclude, I argue that belief heterogeneity of households is important. Empirically, I show that low MPC and high EIS households adjust their macroeconomic expectations more rapidly in response to the business cycle. Through a heterogeneous-agent incomplete information model, I show that this difference is quantitatively important as information allocation has a huge impact to amplification of demand shocks as well as consumption heterogeneity.

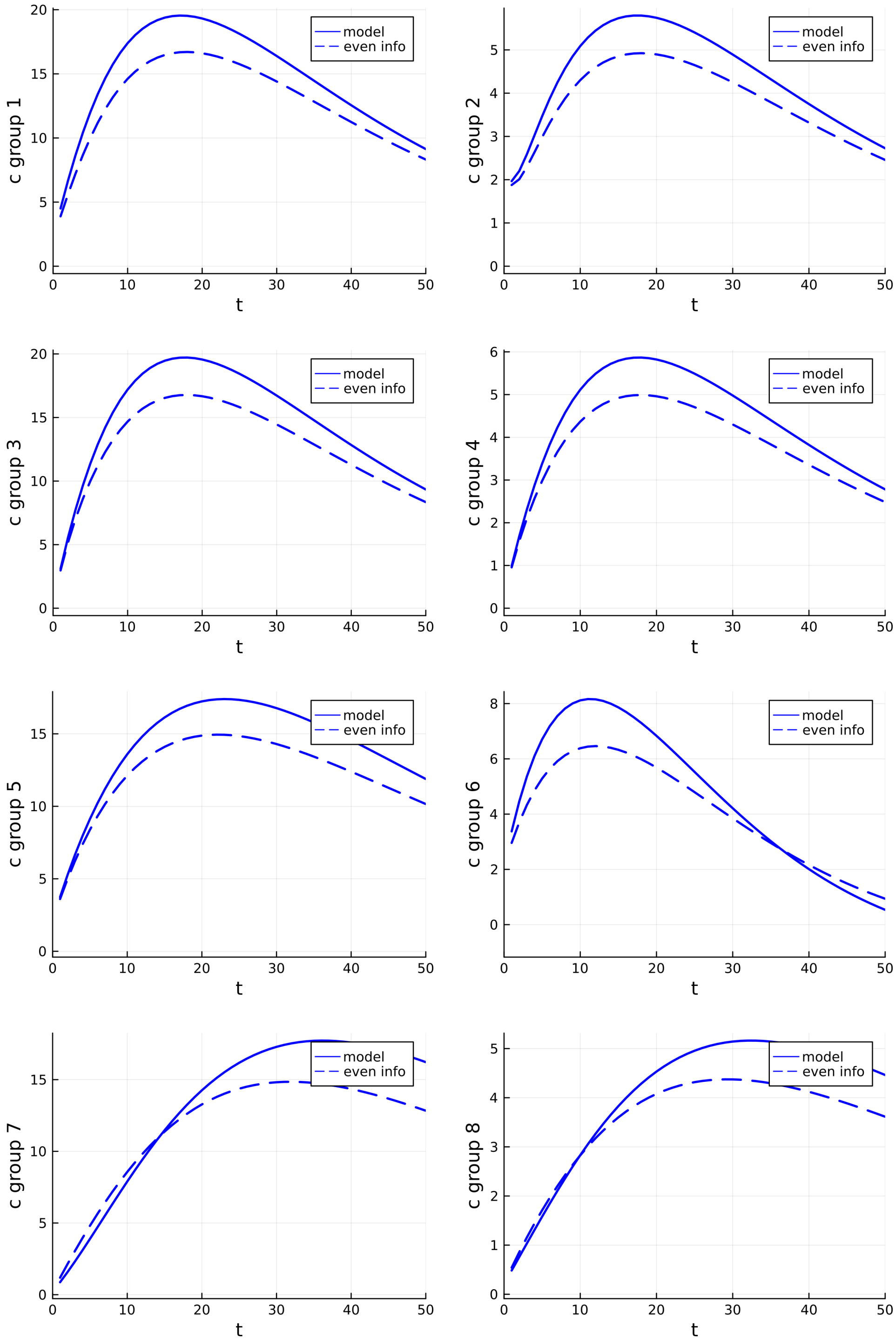


Figure 22: Consumption response for all eight groups

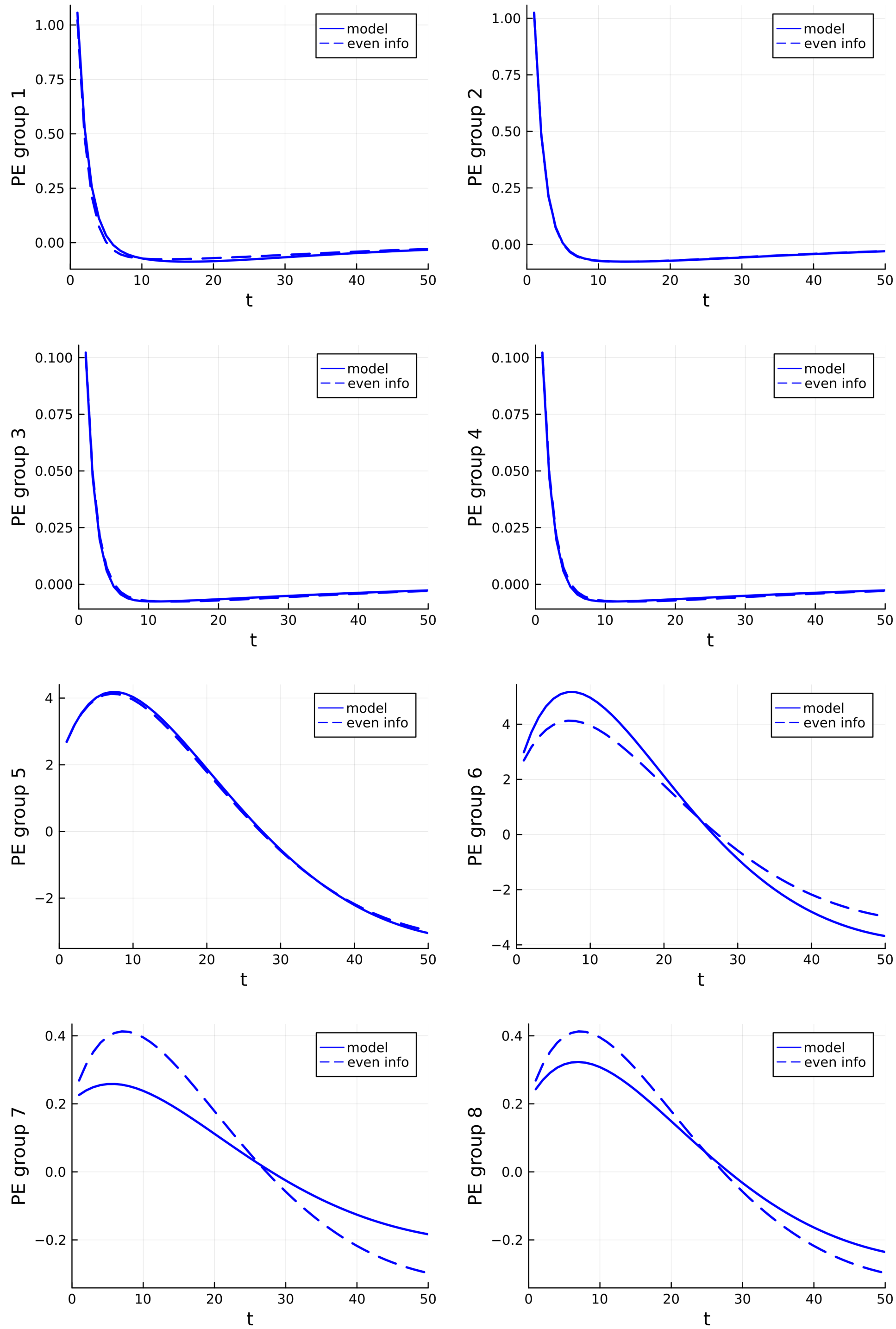


Figure 23: Direct effect for all eight groups

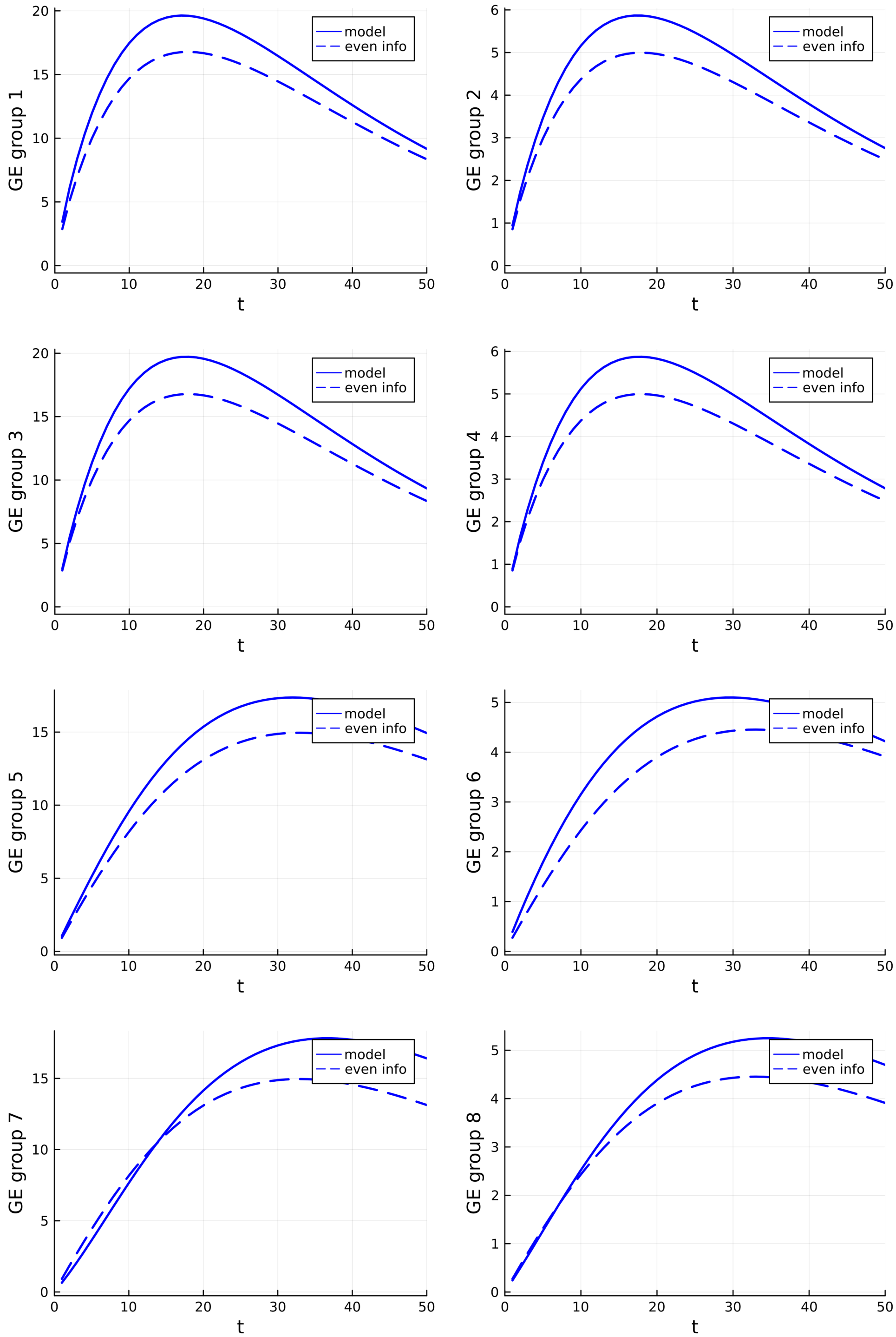


Figure 24: Indirect effect for all eight groups

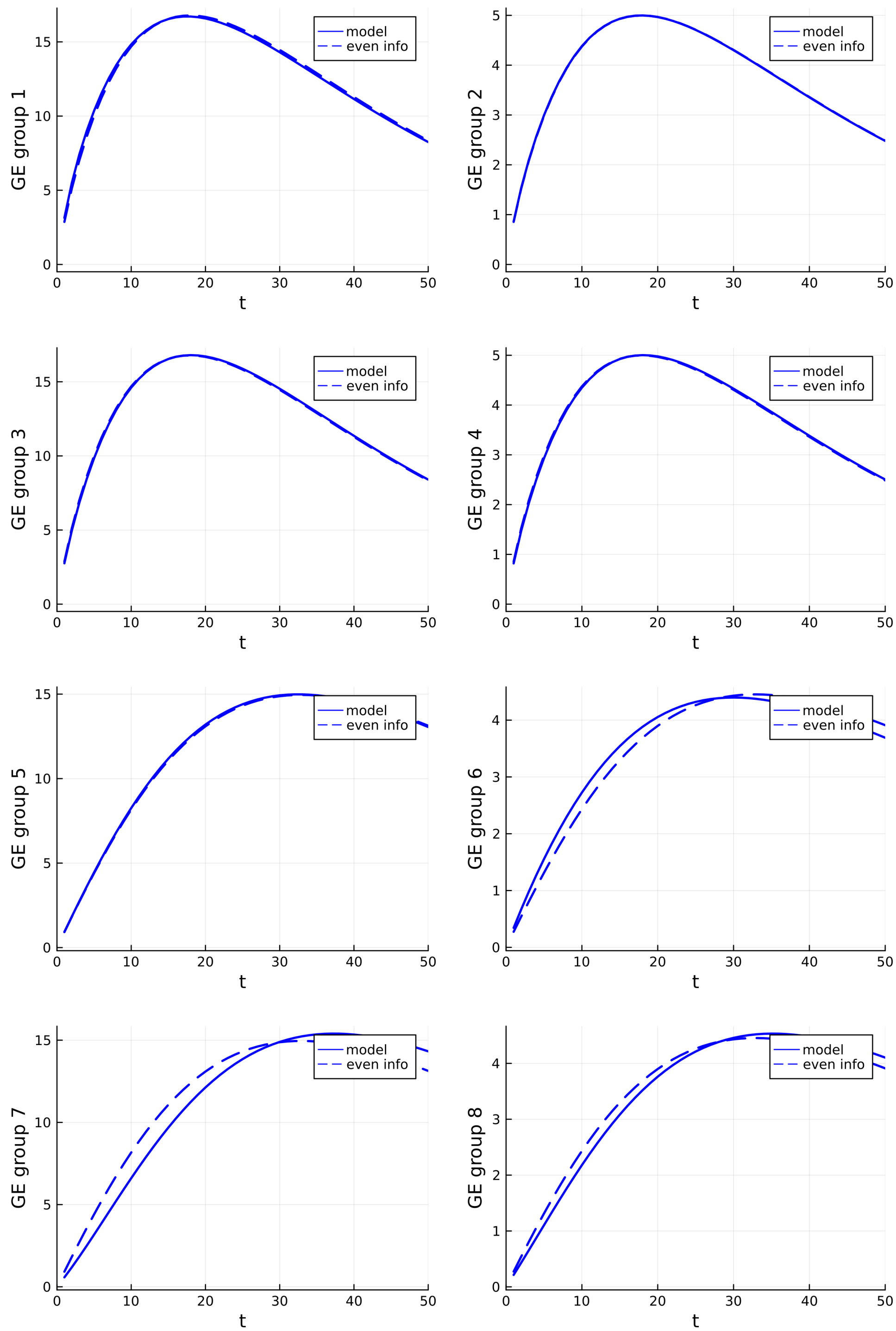


Figure 25: Indirect effect for all eight groups (same $\boldsymbol{h}_y$)

A Log-linearization

This section documents the detail of log-linearizing the household problem. The derivation is similar to the one in Bilbiie (2020).

We begin by deriving its deterministic steady state without any shock. Let the Lagrange multiplier on the budget constraint to be $(\chi\omega_g)^{t-\tau}\mu_t$. The FOCs are

$$(1/\sigma_g + 1)\Phi_{i,t}(C_{i,g,\tau;t})^{1/\sigma_g} = \mu_t$$
$$-\mu_t + \frac{R}{\omega_g}\mu_{t+1}\chi\omega_g = 0$$

This implies the usual Euler equation of

$$(C_{i,g,\tau;t})^{1/\sigma_g} = R\chi\frac{\Phi_{i,t+1}}{\Phi_{i,t}}(C_{i,g,\tau;t+1})^{1/\sigma_g}$$

In the steady state of $R_t\chi = 1$ and $\Phi_t = 1$. The consumption is simply constant.

We log-linearize the solution around a steady state $\chi_t R = 1$ and $C_t = Y_t$. For the budget constraint

$$dC_{i,g,\tau;t} + dS_{i,g,\tau;t} = \frac{R}{\omega_g}dS_{i,g,\tau;t-1} + \lambda_g(Y_t)^{\lambda_g-1}dY_t$$
$$c_{i,g,\tau;t} + s_{i,g,\tau;t} = \frac{1}{\chi\omega_g}s_{i,g,\tau;t-1} + \lambda_g Y^{\lambda_g-1}y_t$$
$$c_{i,g,\tau;t} + s_{i,g,\tau;t} = \frac{1}{\chi\omega_g}s_{i,g,\tau;t-1} + \lambda_g y_t + \epsilon^y_{i,t}$$

where $c_{i,g,\tau;t}$ is the log-derivation from the steady state of $C_{i,g,\tau;t}$ and $s_{i,g,\tau;t}$ is the absolute derivation from the steady state $dS_{i,g,\tau;t}/Y$.

The budget constraint holds for all t. Thus, the life-time budget constraint is given by

$$c_{i,g,\tau,t} + \sum_{k=1}^{\infty}(\chi\omega_g)^k E_t[c_{i,g,\tau;t+k}] = \frac{1}{\chi\omega_g}s_{i,g,\tau;t-1} + \lambda_g y_t + \epsilon^y_{i,t} + \sum_{k=1}^{\infty}(\chi\omega_g)^k\lambda_g E_t[y_{i,t+k}]$$

For the Euler equation,

$$
\begin{aligned}
(C_{i,g,\tau;t})^{1/\sigma_g} &= R\chi \frac{\Phi_{i,t+1}}{\Phi_{i,t}} C_{i,g,\tau;t+1}^{1/\sigma_g} \\
\implies \log(C_{i,g,\tau;t}) &= \log(R\chi) + \sigma_g \log(\frac{\Phi_{i,t+1}}{\Phi_{i,t}}) + \log(C_{i,g,\tau;t+1}) \\
\implies E_{i,g,\tau;t}[c_{i,g,\tau;t+1}] &= c_{i,g,\tau;t} + \sigma_g \phi_{i,t} \\
\implies E_{i,g,\tau;t}[c_{i,g,\tau;t+k}] &= c_{i,g,\tau;t} + \sigma_g \phi_{i,t} + \sigma_g \sum_{j=1}^{k-1} E_{i,g,\tau;t}[\phi_{t+j}]
\end{aligned}
$$

where ϕ_{imt} is the log deviation of $\frac{\Phi_{i,t+1}}{\Phi_{i,t}}$.

Combining the two equations together to obtain the optimal consumption function

$$
\begin{aligned}
\frac{1}{1-\chi\omega_g} c_{i,g,\tau;t} + \sigma_g \frac{\chi\omega_g}{1-\chi\omega_g}\phi_t + \sigma_g \sum_{k=1}^{\infty} \frac{(\chi\omega_g)^{k+1}}{1-\chi\omega_g} E_{i,g,\tau;t}[\phi_{t+k}] &= \frac{1}{\chi\omega_g} s_{i,g,\tau;t-1} + \lambda_g y_t + \sum_{k=1}^{\infty} (\chi\omega_g)^k \lambda_g E_{i,g,\tau} \\
\implies c_{i,g,\tau;t} = & \frac{1-\chi\omega_g}{\chi\omega_g} s_{i,g,\tau;t-1} - \chi\omega_g\sigma_g \big[\phi_{i,t} + \sum_{k=1}^{\infty} (\chi\omega_g)^k E_{i,g,\tau;t}[\phi_{i,t+k}]\big] \\
& + (1-\chi\omega_g)\lambda_g \big[y_{i,t} + \sum_{k=1}^{\infty} (\chi\omega_g)^k E_{i,g,\tau;t}[y_{i,t+k}]\big]
\end{aligned}
$$

The average consumption of group g in period t is given by

$$
c_{g,t} = (1-\omega_g) \sum_{j=0}^{\infty} (\omega_g)^j \int c_{i,g,t-j,t} di
$$

and the total annuity held by the end of the period $t-1$ is given by

$$
(1-\omega_g) \sum_{j=0}^{\infty} (\omega_g)^j s_{g,t-1-j,t-1}
$$

which has only ω_g of them remain in the next period. Thus, the aggregate annuity held by group g at the beginning of t is

$$\omega_g s_{g,t-1} = (1-\omega_g)\sum_{j=0}^{\infty}(\omega_g)^j s_{g,t-1-j,t-1} \tag{27}$$

Let $\beta_g = \chi\omega_g$ be the effective discount factor. The optimal consumption function is

$$c_{g,t} = (1-\beta_g)Rs_{g,t-1} - \beta_g\sigma_g\big[\phi_t + \sum_{k=1}^{\infty}(\beta_g)^k\bar{E}_{g,t}[\phi_{t+k}]\big] + (1-\beta_g)\lambda_g\big[y_t + \sum_{k=1}^{\infty}(\beta_g)^k\bar{E}_{g,t}[y_{t+k}]\big] \tag{28}$$

The group level budget constraint is given by

$$c_{g,t} + s_{g,t} = \frac{1}{\chi}s_{g,t-1} + \lambda_g y_t$$

B Deriving the IRFs

Guess that the solution of the model is an $MA(\infty)$ process of η_t. Specifically,

$$y_t = h_y(L)\eta_t = h_{y,0}\eta_t + h_{y,1}\eta_{t-1} + h_{y,2}\eta_{t-2} + ... \tag{29}$$

Notice that the Impulse Response Function for a one-time shock η_t is IRF$(s) = h_s$. Next step is to construct the IRF for consumption.

B.1 IRF for $c_{g,t}$

For $t = 0, 1, 2, \ldots$

$$\begin{aligned}\frac{dc_{g,t}}{d\eta_0} =& (1-\beta_g)R\frac{ds_{g,t-1}}{d\eta_0}\\ & -\beta_g\sigma_g\Big[\frac{d\phi_t}{d\eta_0}+\sum_{k=1}^{\infty}(\beta_g)^k\frac{d\bar{E}_{g,t}[\phi_{t+k}]}{d\eta_0}\Big]\\ & +(1-\beta_g)\lambda_g\Big[\frac{dy_t}{d\eta_0}+\sum_{k=1}^{\infty}(\beta_g)^k\frac{d\bar{E}_{g,t}[y_{t+k}]}{d\eta_0}\Big]\end{aligned}$$

Collecting the IRF of the consumption of group g gives

$$\boldsymbol{h}_{c,g} = (1-\beta_g)R\boldsymbol{L}\boldsymbol{h}_{s,g} - \beta_g\sigma_g(\boldsymbol{h}_\phi + \begin{bmatrix} m_g'\boldsymbol{E}^\phi_{g,1} \\ m_g'\boldsymbol{E}^\phi_{g,2} \\ \vdots \end{bmatrix}) + (1-\beta_g)\lambda_g(\boldsymbol{h}_y + \begin{bmatrix} m_g'\boldsymbol{E}^y_{g,1} \\ m_g'\boldsymbol{E}^y_{g,2} \\ \vdots \end{bmatrix})$$

where $\boldsymbol{h}_{c,g}$ is the IRF of the consumption for group g, $\boldsymbol{h}_{c,g} = \begin{bmatrix} \frac{dc_{g,0}}{d\eta_0} & \frac{dc_{g,1}}{d\eta_0} & \frac{dc_{g,2}}{d\eta_0} & \ldots \end{bmatrix}.$, $\boldsymbol{h}_\phi = \begin{bmatrix} \frac{d\phi_0}{d\eta_0} & \frac{d\phi_1}{d\eta_0} & \frac{d\phi_2}{d\eta_0} & \ldots \end{bmatrix}$, $\boldsymbol{h}_y$ is the IRF of the output, $\boldsymbol{h}_y = \begin{bmatrix} \frac{dy_0}{d\eta_0} & \frac{dy_1}{d\eta_0} & \frac{dy_2}{d\eta_0} & \ldots \end{bmatrix}$, and $\boldsymbol{h}_{s,g}$ is the IRF for the saving in group g. $\boldsymbol{h}_{s,g} = \begin{bmatrix} \frac{ds_{g,0}}{d\eta_0} & \frac{ds_{g,1}}{d\eta_0} & \frac{ds_{g,2}}{d\eta_0} & \ldots \end{bmatrix}$. $\boldsymbol{L}$ is the lag operator in matrix form $\boldsymbol{L} = \begin{bmatrix} 0_{1\times\infty} \\ I \end{bmatrix}$

The $\boldsymbol{E}^\phi_{g,t}$ and $\boldsymbol{E}^y_{g,t}$ are the IRF for the average forecast of the future ϕ_{t+k} and y_{t+k} repectively using the information at time t.

$$\boldsymbol{E}^\phi_{g,t} = \begin{bmatrix} \frac{d\bar{E}_{g,t}[\phi_{t+1}]}{d\eta_0} & \frac{d\bar{E}_{g,t}[\phi_{t+2}]}{d\eta_0} & \ldots \end{bmatrix}'$$
$$\boldsymbol{E}^y_{g,t} = \begin{bmatrix} \frac{d\bar{E}_{g,t}[y_{t+1}]}{d\eta_0} & \frac{d\bar{E}_{g,t}[y_{t+2}]}{d\eta_0} & \ldots \end{bmatrix}'$$

Finally, the forecast at each horizon is weighted by the proper discount factors given by

$$m'_g = \begin{bmatrix} \beta_g & \beta_g^2 & \beta_g^3 & \dots \end{bmatrix}$$

B.2 Expectation Vector

Since we assume y_t and ϕ_t to be an $MA(\infty)$ process of η_t, we can express $\boldsymbol{E}^y_{g,t}$ in terms of $\boldsymbol{h}_y$. For $k \geq 0$

$$E_{i,g,t}[y_{t+k}] = h_0 \underbrace{E_{i,g,t}[\eta_{t+k}]}_{=0} + h_1 \underbrace{E_{i,g,t}[\eta_{t+k-1}]}_{=0} + \ldots + h_k E_{i,g,t}[\eta_t] + h_{k+1} E_{i,g,t}[\eta_{t-1}] + \ldots$$

Taking average and differentiating with η_0 yield

$$\frac{d\bar{E}_{g,t}[y_{t+k}]}{d\eta_0} = h_k \frac{d\bar{E}_{g,t}[\eta_t]}{d\eta_0} + h_{k+1} \frac{d\bar{E}_{g,t}[\eta_{t-1}]}{d\eta_0} + \ldots$$

The expectation vector $\boldsymbol{E}^y_{g,t}$ can be written as

$$\boldsymbol{E}^y_t \equiv \begin{bmatrix} \frac{d\bar{E}_t[y_{t+1}]}{d\eta_0} \\ \frac{d\bar{E}_t[y_{t+2}]}{d\eta_0} \\ \vdots \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & \frac{d\bar{E}_t[\eta_t]}{d\eta_0} & \frac{d\bar{E}_t[\eta_{t-1}]}{d\eta_0} & \dots \\ 0 & 0 & \frac{d\bar{E}_t[\eta_t]}{d\eta_s} & \dots \\ \vdots & \vdots & \vdots & \dots \end{bmatrix}}_{\equiv \boldsymbol{M}^\eta_t} \boldsymbol{h}_y$$

where $\boldsymbol{M}^\eta_t$ captures the IRF to the expectation of all the past η_t. To compute $\boldsymbol{M}^\eta_t$, one can use a standard Wiener-Hopf filter

$$\bar{E}_{g,t}[\eta_{t-k}] = \frac{\gamma_g \tau_{g,u}}{\rho \tau_\eta}(L^k + \gamma_g L^{k-1} + \cdots + \gamma_g^{k-1} L + \gamma_g^k)(1 + \gamma_g L + \gamma_g^2 L^2 + \cdots)\eta_t \tag{30}$$

where $\gamma_g = \frac{1}{2}\left[\rho + \frac{1}{\rho}(1 + \tau^x_g/\tau^\phi) - \sqrt{(\rho + \frac{1}{\rho}(1 + \tau^x_g/\tau^\phi))^2 - 4}\right]$. The coefficients of the process in 30 represent the IRF for the $\bar{E}_{g,t}[\eta_{t-k}]$, which can be used to fill up $\boldsymbol{M}^\eta_t$

To simplify the notation, we let

$$\boldsymbol{W}_g \equiv \begin{bmatrix} m_g'\boldsymbol{M}_0^\eta \\ m_g'\boldsymbol{M}_1^\eta \\ \vdots \end{bmatrix}$$

B.3 Final System

The consumption IRF becomes

$$\boldsymbol{h}_{c,g} = (1-\beta_g)R\boldsymbol{L}\boldsymbol{h}_{s,g} - \beta_g\sigma_g(\boldsymbol{h}_\phi + \boldsymbol{W}_g\boldsymbol{h}_\phi) + (1-\beta_g)\lambda_g(\boldsymbol{h}_y + \boldsymbol{W}_g\boldsymbol{h}_y)$$

The IRF of the group level budget constraint and the market clearing condition comes from taking the derivative with respect to η_0. Collecting all of them into a vector yields

$$\boldsymbol{h}_{c,g} + \boldsymbol{h}_{s,g} = R\boldsymbol{L}\boldsymbol{h}_{s,g} + \lambda_g\boldsymbol{h}_y$$

and

$$\boldsymbol{h}_y = \sum_g \pi_g\boldsymbol{h}_{c,g}$$

B.4 Solving the IRF for y_t

The IRF for the wealth is the compounded IRF of saving

$$\boldsymbol{h}_{s,g} = (1-R\boldsymbol{L})^{-1}(\lambda_g\boldsymbol{h}_y - \boldsymbol{h}_{c,g})$$

$$\boldsymbol{h}_{c,g} = (1-\beta_g)\underbrace{R\boldsymbol{L}(1-R\boldsymbol{L})^{-1}}_{\equiv \boldsymbol{A}}(\lambda_g \boldsymbol{h}_y - \boldsymbol{h}_{c,g}) - \beta_g\sigma_g(\boldsymbol{h}_\phi + W_g\boldsymbol{h}_\phi) + (1-\beta_g)\lambda_g(\boldsymbol{h}_y + W_g\boldsymbol{h}_y)$$

$$(\boldsymbol{I} + (1-\beta_g)\boldsymbol{A})\boldsymbol{h}_{c,g} = -\beta_g\sigma_g(\boldsymbol{I}+\boldsymbol{W}_g)\boldsymbol{h}_\phi + (1-\beta_g)\lambda_g(\boldsymbol{A}+\boldsymbol{I}+\boldsymbol{W}_g)\boldsymbol{h}_y$$

$$\boldsymbol{h}_{c,g} = -\beta_g\sigma_g\boldsymbol{B}_g(\boldsymbol{I}+\boldsymbol{W}_g)\boldsymbol{h}_\phi + (1-\beta_g)\lambda_g\boldsymbol{B}_g(\boldsymbol{A}+\boldsymbol{I}+\boldsymbol{W}_g)\boldsymbol{h}_y$$

with $\boldsymbol{B}_g = (\boldsymbol{I} + (1-\beta_g)\boldsymbol{A})^{-1}$. Let $\boldsymbol{M}_\phi = \sum_g^G \pi_g(-\beta_g\sigma_g\boldsymbol{B}_g(\boldsymbol{I}+\boldsymbol{W}_g))$ and $\boldsymbol{M}_y = \sum_g^G \pi_g(1-\beta_g)\lambda_g\boldsymbol{B}_g(\boldsymbol{A}+\boldsymbol{I}+\boldsymbol{W}_g)$ Now, we have a system for pinning down the $\boldsymbol{h}_y$

$$\boldsymbol{h}_y = \sum_g^G \pi_g\boldsymbol{h}_{c,g} = \boldsymbol{M}_\phi\boldsymbol{h}_\phi + \boldsymbol{M}_y\boldsymbol{h}_y$$

B.5 Calculating the Theoretical Regression Coefficient

C Derivation for Comparative Statics

Derive the comparative statics for the information allocation parameter α

$$\begin{aligned}\frac{\partial \boldsymbol{h}_y}{\partial\alpha} &= -(\boldsymbol{I}-\boldsymbol{M}_y)^{-1}\frac{\partial(\boldsymbol{I}-\boldsymbol{M}_y)}{\partial\alpha}(\boldsymbol{I}-\boldsymbol{M}_y)^{-1}\boldsymbol{M}_\phi\boldsymbol{h}_\phi + (\boldsymbol{I}-\boldsymbol{M}_y)^{-1}\frac{\partial\boldsymbol{M}_\phi}{\partial\alpha}\boldsymbol{h}_\phi \\ &= (\boldsymbol{I}-\boldsymbol{M}_y)^{-1}(\frac{\partial\boldsymbol{M}_y}{\partial\alpha}\boldsymbol{h}_y + \frac{\partial\boldsymbol{M}_\phi}{\partial\alpha}\boldsymbol{h}_\phi)\end{aligned}$$

The two derivatives are given by

$$\frac{\partial\boldsymbol{M}_y}{\partial\alpha} = \frac{\partial}{\partial\alpha}\left[\sum_g \pi_g(1-\beta_g)\lambda_g\boldsymbol{B}_g(\boldsymbol{A}+\boldsymbol{I}+\boldsymbol{W}_g)\right] = \sum_g \pi_g(1-\beta_g)\lambda_g\boldsymbol{B}_g\frac{\partial\boldsymbol{W}_g}{\partial\alpha}$$

$$\frac{\partial\boldsymbol{M}_\phi}{\partial\alpha} = \frac{\partial}{\partial\alpha}\left[-\sum_g \sigma_g\pi_g\beta_g\boldsymbol{B}_g(\boldsymbol{I}+\boldsymbol{W}_g)\right] = -\sum_g \pi_g\sigma_g\beta_g\boldsymbol{B}_g\frac{\partial\boldsymbol{W}_g}{\partial\alpha}$$

D Data Description

More details for the data part

E Forecast Formation

Individual i in group g has a prior on a macroeconomic variable y_t, which follows

$$y_t \sim N(\mu_{g,t}, (\tau_0)^{-1})$$

$\mu_{g,t}$ is the subjective mean of beliefs about the macroeconomic variable. The mean of y_t observed by the econometricians is assumed to be zero. The $\mu_{g,t}$ is assumed to be independent of y_t. There is a signal for the macroeconomic variable, $x_{i,t}$, which is a noisy signal of the true value of y_t. The signal follows

$$x_{i,g,t} = y_t + \epsilon_{i,g,t}$$

with

$$\epsilon_{i,g,t} \sim N(0, (\tau_{g,t})^{-1})$$

Thus, $\tau_{g,t}$ measures the precision of the signal. We assume y_t and $\tau_{g,t}$ to be independent. Thus, the posterior mean of y_t is given by

$$E_{i,g,t}[y_t] = \lambda_{g,t} x_{i,g,t} + (1 - \lambda_{g,t})\mu_{g,t} \tag{31}$$

with $\lambda_{g,t} = \frac{\tau_{g,t}/\tau_0}{1+\tau_{g,t}/\tau_0}$

The average forecast of the group is given by

$$\bar{E}_{g,t}[y_t] = \lambda_{g,t} y_t + (1 - \lambda_{g,t})\mu_{g,t} \tag{32}$$

which can be rewritten as

$$\bar{E}_{g,t}[y_t] = \bar{a}_g + \bar{\lambda}_g y_t + e_{g,t} \tag{33}$$

where $\bar{\lambda}_g = \int \lambda_{g,t} dt$ and $\bar{\mu}_g = \int \mu_{g,t} dt$, $\bar{a}_g = -(1-\bar{\lambda}_g)\bar{\mu}_g$, $e_{g,t} = (\lambda_{g,t} - \bar{\lambda}_g)y_t - (\lambda_{g,t} - \bar{\lambda}_g)\bar{\mu}_g + (1-\bar{\lambda}_g)(\mu_{g,t} - \bar{\mu}_g) - (\lambda_{g,t} - \bar{\lambda}_g)(\mu_{g,t} - \bar{\mu}_g)$. As long as $\mu_{g,t}$, $\tau_{g,t}$ and y_t are independent, $Cov(y_t, e_t) = 0$. This implies that the average precision can be estimated by regressing the forecast on the realized value of the macroeconomic variable.

F Estimating average beliefs using Survey Data

The Michigan Survey records only the qualitative responses, “increase", “no change" or “decrease". I extended the method in Carlson and Parkin (1975), Mankiw *et al.* (2003) and Bhandari *et al.* (2019) to estimate the average quantitative belief of each subgroup.

Individual i in group g makes a forecast $y^f_{i,g,t}$ about an macroeconomic variable y_t. The forecast $y^f_{i,g,t}$ follows a distribution

$$y^f_{i,g,t} \sim N(\mu_{g,t}, \sigma^2_{g,t})$$

Our goal is to estimate the group average forecast, $\mu_{g,t}$ and use it to study the relationship with y_t.

Following the assumptions in the literature, I assume that households answer "increase" or "decrease" only when their forecast exceeds the thresholds. Let $y^*_{i,g,t}$ be the response to the survey

$$y^*_{i,g,t} = \begin{cases} \text{“increase"} & \text{if } y^f_{i,g,t} > a \\ \text{“no change"} & \text{if } -a \leq y^f_{i,g,t} \leq a \\ \text{“decrease"} & \text{if } y^f_{i,g,t} < -a \end{cases}$$

Thus,

$$P(y^*_{i,g,t} = \text{“increase"}) = 1 - \Phi(\frac{a - \mu_{g,t}}{\sigma^2_{g,t}})$$

and

$$P(y^*_{i,g,t} = \text{“decrease"}) = \Phi(\frac{-a - \mu_{g,t}}{\sigma^2_{g,t}})$$

We have two data point to pin down $\{a, \mu_{g,t}, \sigma_{g,t}\}$ so we still need one more data point. Both SPF and Michigan Survey asked the respondents to give a quantitative forecast for inflation. In addition, forecasters in SPF were asked to forecast other macroeconomic variables, such as unemployment. Following Bhandari *et al.* (2019), we assume that the ratio of cross-sectional dispersion of forecasts between the survey respondents in SPF and Michigan Survey is constant across other macroeconomic variables. This allows us to infer the overall dispersion of the forecasts in Michigan Survey. a is picked to match this value.

G Forecast by Groups

G.1 Time Series of Forecasts by Groups

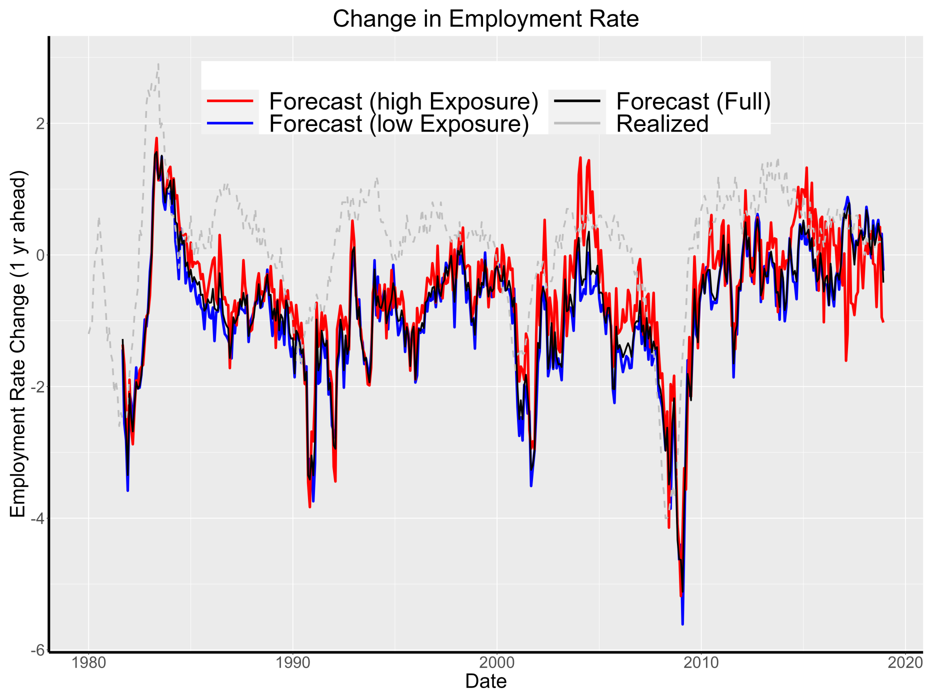


Figure 26: Forecasts by Exposures group

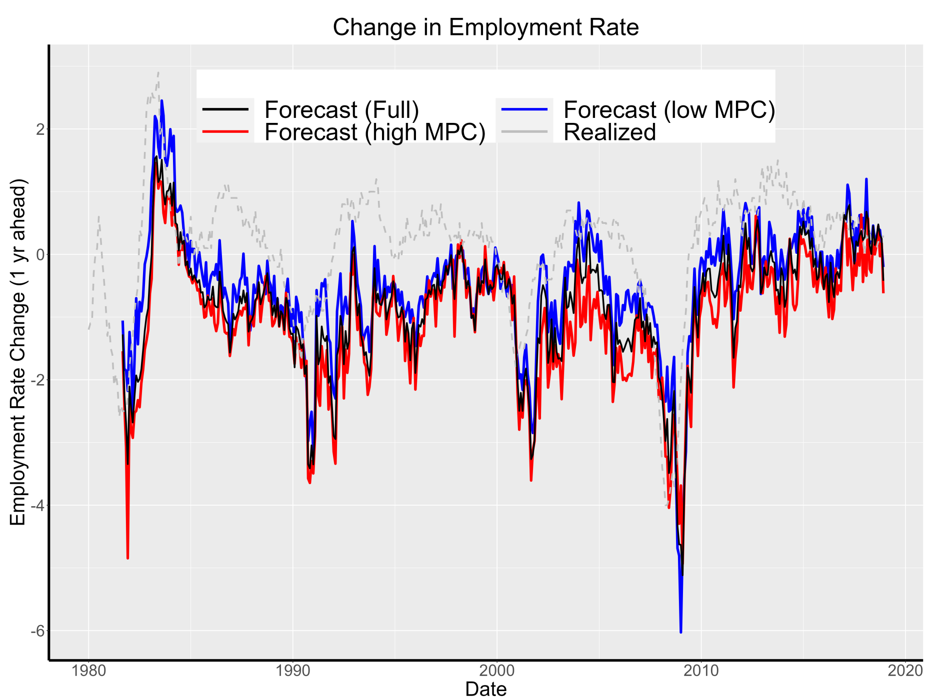


Figure 27: Forecasts by MPCs group

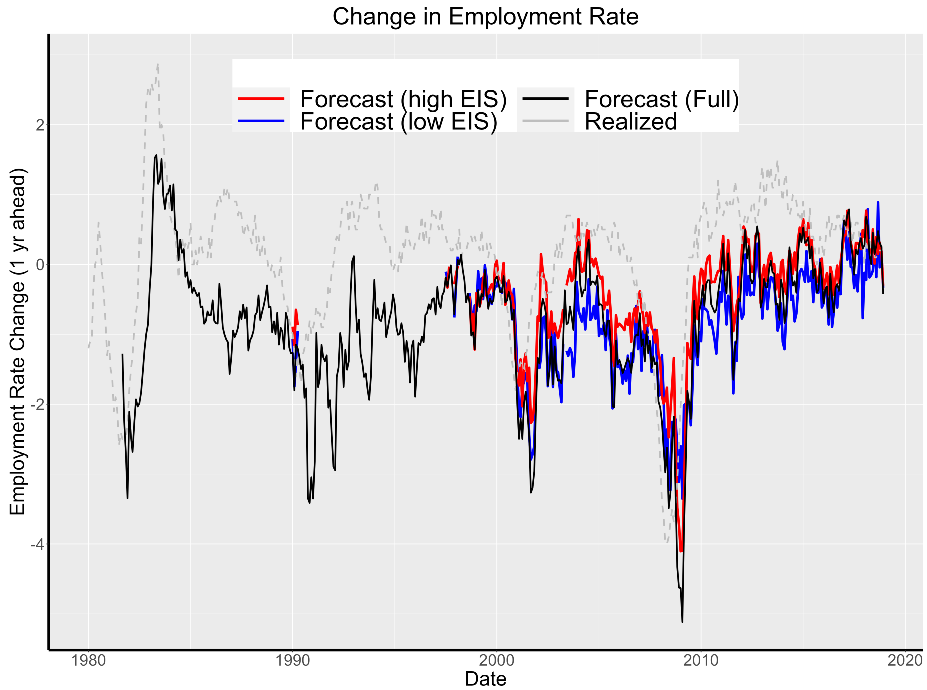


Figure 28: Forecasts by EISs (Proxy by Stock Market Participation) group

G.2 Regression of Forecasts by Groups

Table 3: Forecast by Exposures on Realized Employment Change

	Dependent variable:		
	Full	High Exposure	Low Exposure
	(1)	(2)	(3)
Realized	0.278***	0.264***	0.286***
	(0.028)	(0.031)	(0.029)
Constant	−0.760***	−0.563***	−0.817***
	(0.043)	(0.047)	(0.044)
Observations	490	490	490
R^2	0.164	0.129	0.166
Adjusted R^2	0.162	0.128	0.165
Residual Std. Error (df = 488)	0.943	1.031	0.965
F Statistic (df = 1; 488)	95.790***	72.536***	97.364***
Note:			*p<0.1; **p<0.05; ***p<0.01

Table 4: Forecast by MPCs on Realized Employment Change

	Dependent variable:		
	Full	High MPC	Low MPC
	(1)	(2)	(3)
Realized	0.278***	0.247***	0.296***
	(0.028)	(0.029)	(0.028)
Constant	−0.760***	−0.978***	−0.442***
	(0.043)	(0.043)	(0.042)
Observations	490	490	490
R^2	0.164	0.131	0.190
Adjusted R^2	0.162	0.130	0.189
Residual Std. Error (df = 488)	0.943	0.956	0.917
F Statistic (df = 1; 488)	95.790***	73.748***	114.763***
Note:			*p<0.1; **p<0.05; ***p<0.01

Table 5: Forecast by EISs on Realized Employment Change

	Dependent variable:		
	Full	High EIS	Low EIS
	(1)	(2)	(3)
Realized	0.278***	0.202***	0.130***
	(0.028)	(0.026)	(0.025)
Constant	−0.760***	−0.337***	−0.748***
	(0.043)	(0.046)	(0.045)
Observations	490	295	295
R^2	0.164	0.171	0.084
Adjusted R^2	0.162	0.168	0.081
Residual Std. Error	0.943 (df = 488)	0.797 (df = 293)	0.766 (df = 293)
F Statistic	95.790*** (df = 1; 488)	60.401*** (df = 1; 293)	27.032*** (df = 1; 293)
Note:			*p<0.1; **p<0.05; ***p<0.01

References

ANGELETOS, G.-M. and HUO, Z. (2021). Myopia and Anchoring. *American Economic Review*, **111** (4), 1166–1200.

— and LIAN, C. (2018). Forward Guidance without Common Knowledge. *American Economic Review*, **108** (9), 2477–2512.

— and — (2022). Dampening General Equilibrium: Incomplete Information and Bounded Rationality. *Working Paper*.

AUCLERT, A., ROGNLIE, M. and STRAUB, L. (2018). The Intertemporal Keynesian Cross. *Working Paper*, p. w25020.

—, — and — (2020). Micro Jumps, Macro Humps: Monetary Policy and Business Cycles in an Estimated HANK Model. *Working Paper*.

BHANDARI, A., BOROVICKA, J. and HO, P. (2019). Survey Data and Subjective Beliefs in Business Cycle Models. *Working Paper*, **19** (14), 1–60.

BILBIIE, F. O. (2020). The New Keynesian cross. *Journal of Monetary Economics*, **114**, 90–108.

BLANCHARD, O. J. (1985). Debt, Deficits, and Finite Horizons. *Journal of Political Economy*, **93** (2), 223–247, publisher: University of Chicago Press.

CARLSON, J. A. and PARKIN, M. (1975). Inflation Expectations. *Economica*, **42** (166), 123–138, publisher: [London School of Economics, Wiley, London School of Economics and Political Science, Suntory and Toyota International Centres for Economics and Related Disciplines].

DAS, S., KUHNEN, C. M. and NAGEL, S. (2020). Socioeconomic Status and Macroeconomic Expectations. *The Review of Financial Studies*, **33** (1), 395–432.

FRIEDMAN, M. (1957). *A Theory of the Consumption Function*. Princeton University Press, backup Publisher: National Bureau of Economic Research Type: Book.

GUERREIRO, J. (2022). Belief Disagreement and Business Cycles. *Working Paper*, p. 65.

GUVENEN, F. (2006). Reconciling conflicting evidence on the elasticity of intertemporal substitution: A macroeconomic perspective. *Journal of Monetary Economics*, **53** (7), 1451–1472.

MANKIW, N. G., REIS, R. and WOLFERS, J. (2003). Disagreement about Inflation Expectations. *NBER Macroeconomics Annual*, **18**, 209–248, publisher: The University of Chicago Press.

MODIGLIANI, F. (2005). *The Collected Papers of Franco Modigliani*. The MIT Press.

PATTERSON, C. (2023). The Matching Multiplier and the Amplification of Recessions. *American Economic Review*, **113** (4), 982–1012.

YAARI, M. E. (1965). Uncertain Lifetime, Life Insurance, and the Theory of the Consumer. *The Review of Economic Studies*, **32** (2), 137–150, publisher: [Oxford University Press, Review of Economic Studies, Ltd.].